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FLOW ANALYSIS THROUGH INWARD TRAPEZOIDAL CHANNELS

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Abstract: For the first time, a theoretical, validated by experimental investigation on the flow through inward trapezoidal channel is done. A near-linear depth-discharge relationship is obtained by optimization such that the computed discharge is well within 2% deviation from the corresponding theoretical values. It is found that the linear relationship for practical and theoretical are given for z_1 = 0.095 and z_2 =0.123, respectively are $Q_{La} = 2.1725y - 0.3159$, $Q_{Lt} = 2.0171y - 0.1937$ and $Q_{La} = 2.02y - 0.2027$, $Q_{Lt} = 2.0171y - 0.1937$. A geometrically simple flume which can be constructed or existing flume can be modified on-site with straight sides in the shape of inward trapezoidal shape has been theoretically analyzed, graphically and experimentally validated. It is shown that it can be used for normal flows as a near-linear flow measuring device. The flow in the new device is shown to vary linearly with respect to the flow depth valid within a pre-defined range of depth within prefixed error of deviation from the theoretical discharge. The significance is the shift is from measurement of discharge through computations to direct reading and auto-recording of the discharge with a piezometer or a float-regulated device.

Keywords: Open Channel Flow, Flow measurement, Inward Trapezoidal Channel (ITC), Simple linear channels.

1. Introduction and Literature Review

Flow measurement is a field in engineering which, of late, has assumed extreme importance, due to the improved awareness about water management. In most of the cases, for a predetermined discharge, the channel dimensions are designed. Least investigative work is done on the reverse process of determining the flow characteristics from the known channel dimensions. The discharge measurement is a much unpredictable task which depends on multiple factors. The major factor hindering the measurement is being the induced loss of head which reduces the possibility of water reaching the tail-end users of an irrigation channel. Generally, the measuring flumes are designed by introducing a head loss between two sections. The cross-section of a channel may be closed or open at the top. The channels that have an open top are referred to as open channels while those with a closed top and running full are referred to as closed conduits (Macharia Karimi.et.al., 2014)

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Discharge Measurement in open channels is the main concerns in irrigation, environmental and hydraulic engineering field. Flow measuring structures, which are typically used to operate as a control in the channel, provide a special link between the flow discharge and up-stream head (Ismail Aydina.et.al, 2011, Mohamad Reza Madadi.et.al, 2013, 2014, Shesha Prakash and Adarsh, 2023). In particular, while designing sewage treatment plants, the accuracy of the treatment depends on the accuracy of the input discharge prediction and in case of grey water, it is difficult to handle the flow measurement. In the present research work, it is attempted to overcome most of the drawbacks, thereby, even the flow measurement can be automated.

In this paper, a theoretical, graphical and experimental investigation on the flow through inward trapezoidal channel has been done; the main concern in flow measurement in channels is that intrusive devices reduce drastically the head/energy of flow and thereby the flow does not reach the tail end of the channels. In the present research, it is to show that with non-intrusive devices, flow measurement can be done with least interference and without computations with complicated equations. The major thrust in the paper is that, the shift is from measurement of head and thereby computes discharge through equations, to direct recording of the discharge similar to reading the head (similar to rotameters in closed conduits).

Shesha Prakash and Adarsh (April 2023, June 2023) investigated the linear flow characteristics for flow through the rectangular channel, which is highly useful in recording discharges in existing rectangular channels. However, the threshold depth (for which the near linear equation is not valid) of measuring range is high in rectangular channel and most of the discharge in the threshold depth region cannot be measured with the proposed near-linear equation. Hence the analysis with slight inclination of the vertical sides was taken-up which was termed as inward trapezoidal channel (ITC). From a known threshold-depth, it was found that the starting point drastically reduced from that of rectangular channel. Depending upon the geometry of existing channel, for lesser threshold depth and larger range, inward trapezoidal channel will be more suitable. Incidentally it was found that Subramanya (1986) has also investigated theoretically about Trapezoidal channels with negative side slopes and has given monograms and has even produced a graph to show the variation of non-dimensional parameters such as depth of flow and discharge through the channel as shown in Fig. 1. He has used Manning’s equation to find the depth-discharge relationship (Chow. V.T. 1959, Henderson 1967).

Keshava Murthy and Giridhar (1988, 89) attempted new techniques for finding linear flow characteristics for flow through sharp crested weirs. Later, Keshava Murthy and Shesha Prakash (1993, 94, 95) improved and found new faster, better and exact optimization methods to analyse the flow and get the discharge-head (Q - h^n) relationships in the form of linear, quadratic, logarithmic, exponential and for any given power, thereby mastering the technique of optimization. The optimization technique adopted by Keshava Murthy and Shesha Prakash has been used in this paper to exploit the linear flow characteristics of an Inward Trapezoidal Channel (ITC). The sides of the channel being straight, without change in bed levels, not only easy to adopt with existing channels and also induce least error in fabrication resulting in accurate computations in discharge measurements. However, neither detailed analysis nor equation is given; hence the detailed analysis is carried out in this research work.

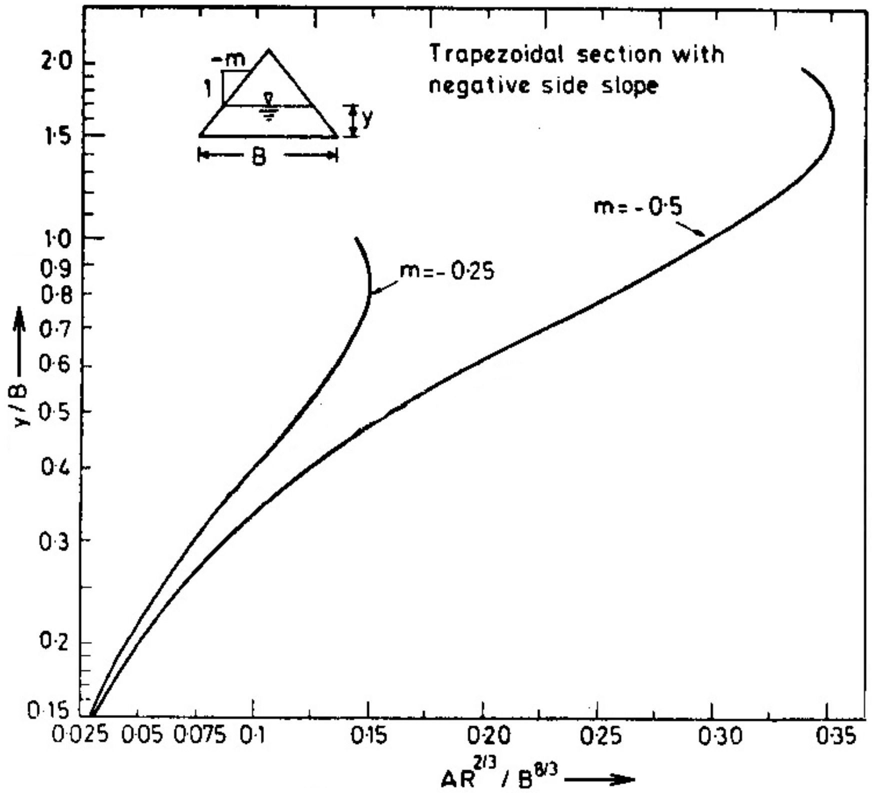


Fig 1: Variation of $\boldsymbol{AR^{2/3}}$ in Channels (Closed Top)

It can be observed from Fig. 1 that the curve straightens with reduction in side slopes (*m*) and this near-linear relationship can be exploited to develop a simple linear equation to measure flow rate with the available flow depth.

In the present research, it is to show that with non-intrusive devices, flow measurement can be done with least interference and without computations through complicated equations.

2. Formulation of the Problem

Consider flow through an inward trapezoidal flume with 2*b* as the bed width, Maximum permissible flow depth *d* and 1:*z* (V:H) inward side slope with a flow depth *y* as shown in Fig. 2. The flow through the given inward trapezoidal open channel can be analysed by two well accepted equations, viz; Chezy's equation and Manning's equation**.** From the previous research work done by the authors (Shesha Prakash and Adarsh, 2023, Publication under review), the non-dimensional equations for flow through ITC is as follows:

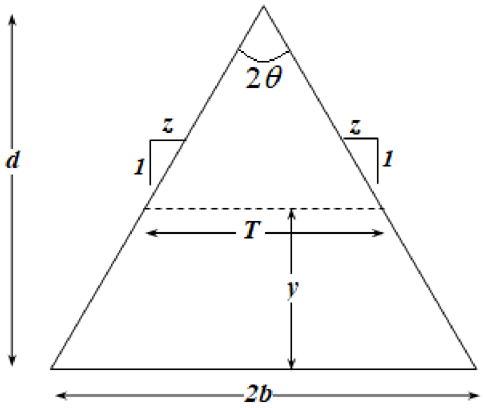


Fig 2: Definition Sketch

2.1 Chezy's Equation

But the dimensions of $C = L^{\frac{1}{2}}T^{-1}$, Hence, the non-dimensional discharge can be expressed as a function of flow depth Y as below.

$$Q = \frac{q}{2^{-\frac{1}{2}} b^{\frac{5}{2}} C\sqrt{S}} = \frac{\left(2Y - Y^2 z\right)^{\frac{3}{2}}}{\left(1 + Y\sqrt{1+z^2}\right)^{\frac{1}{2}}} \tag{2.1}$$

2.2 Manning's equation

$$Q = q\frac{2^{\frac{2}{3}}}{\sqrt{S}}\frac{n}{b^{\frac{8}{3}}} = \frac{\left(2Y - Y^2 z\right)^{\frac{5}{3}}}{\left(1 + Y\sqrt{1+z^2}\right)^{\frac{2}{3}}} \tag{2.2}$$

Which can also expressed in terms of vortex angle of the flume as below:

$$Q = \frac{\left(2Y - Y^2 \tan(\theta)\right)^{\frac{5}{3}}}{\left(1 + Y\sec(\theta)\right)^{\frac{2}{3}}} \tag{2.3}$$

Where C is the Chezy's constant, n is the Manning's constant, S is the water surface slope

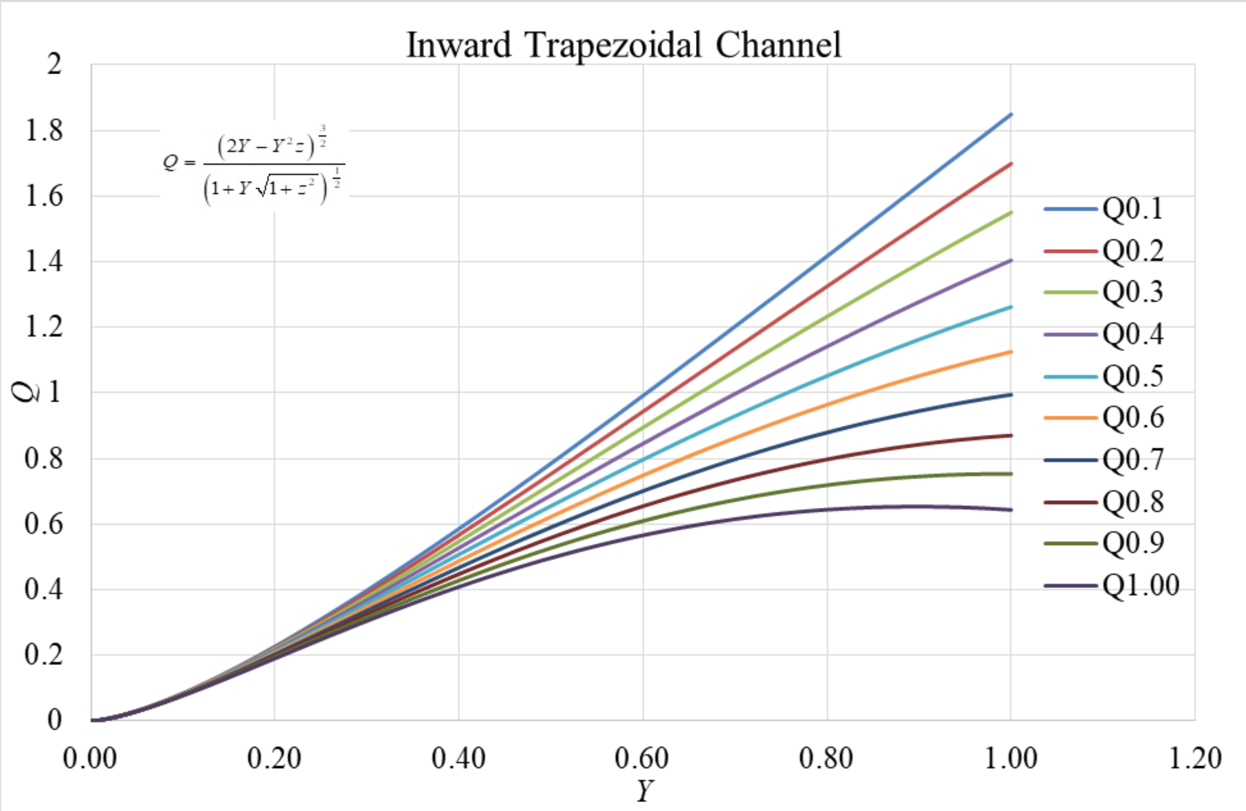


Fig 3: Theoretical Variation of Discharge (Non-dimensional) with Depth (Non-dimensional) (Eq. 2.1)

The theoretical head-discharge curve is shown in Fig 3. It is seen from Fig 3 that the discharge-head relationship is near linear in a certain range of head beyond a certain minimum value for a given value of *z*. This property of the inward trapezoidal channel is exploited to evolve a linear relationship between Q and Y in certain range of head, such that the relative percentage deviation between theoretical discharge and the one given by linear relation does not exceed a prefixed maximum permissible error.

3. Optimization Procedure

Through optimization the following equations have been developed.

Let the proposed near-linear non-dimensional discharge

$$Q_L = mY + Q_c \quad (3.1)$$

be the proposed optimal linear head-discharge relationship (where, m is the constant of proportionality and Q_c is the discharge intercept) to substitute the theoretical head-discharge relationship.

$$Q = f(Y) \quad (3.2)$$

In a certain range. Letting $Q = f(Y)$, $K_u = \left(1 + \frac{E}{100}\right)$ and $K_d = \left(1 - \frac{E}{100}\right)$ where E is the prefixed maximum permissible relative deviation of the proposed linear function and the theoretical head-discharge function. These defined two explicit curves $f_1(Y)$ and $f_2(Y)$ forming the lower and upper bounds for the linear function as shown in figure 4.

$$f_1(Y) = K_u f(Y) \tag{3.3}$$

$$f_2(Y) = K_d f(Y) \tag{3.4}$$

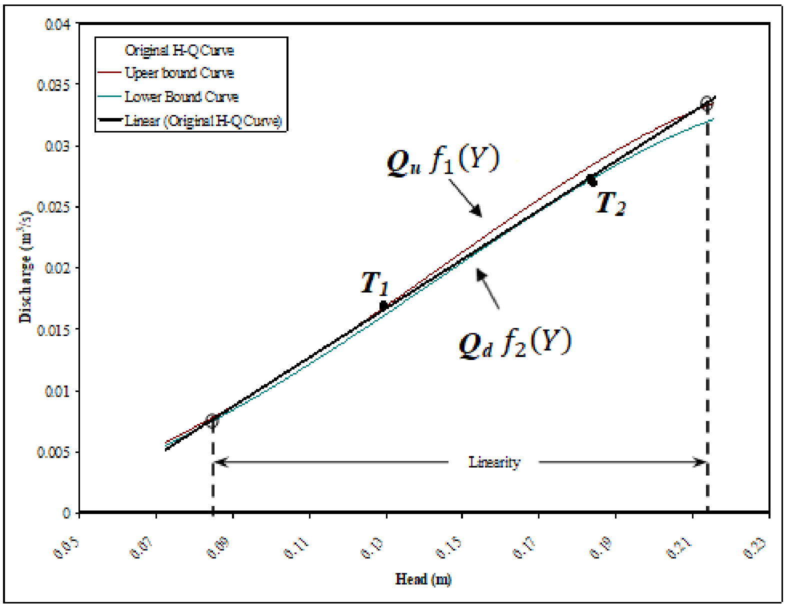


Fig 4: Optimization procedure with upper and lower bound error curves

Further the detailed optimization procedure, table values is mentioned in the previous research work developed by the authors (Shesha Prakash and Adarsh, 2023, Publication under review). The table of flow parameters for various values of *z* is reproduced in the present work and used for experimental validation.

4. Analysis

With the flow parameters for a rectangular channel (Shesha Prakash et.al April 2023, June 2023), the discharge below the proposed measurable near-linear range cannot be measured. To improve the flow characteristics, the analysis for slight inclination of the vertical sides, termed as inward trapezoidal channel (ITC), was considered to provide better measurable linear range from nearer to the bed of the channel. The Table 2 provides flow parameters for various values of inclination of the side (*z*). A typical curve for specific value of *z*=0.25 has been show in Fig. 5, wherein it can be observed that a near straight-line relationship is within the bound curves of Q_u and Q_d. The Staring point *A* (upper limit of the threshold depth) of the measurable near linearity range can also be observed which validates the theoretical analysis and the table values of inward trapezoidal channel. Similarly for various values of *z* the starting point, Ending point (point at which the proposed linear range terminates), linearity range and the corresponding discharge equation is as mentioned in the below Table 2.

Table 2: Inward Trapezoidal Channel Flow Parameters for Various Values of *z*

S.N	z	Y_A	Y_B	LR	m	Q_c	Q_L
1	0.25	0.1935	1.4052	1.2117	1.7252	-0.1263	1.7252(Y – 0.0732)
2	0.50	0.1232	0.8184	0.6952	1.4216	-0.0684	1.4216(Y-0.0481)
3	0.75	0.1219	0.5816	0.4597	1.2337	-0.0441	1.2337(Y-0.0357)
4	1.00	0.0744	0.4503	0.3759	1.1016	-0.0312	1.1016(Y-0.0283)
5	1.25	0.0558	0.3682	0.3124	1.0024	-0.0235	1.0024(Y-0.0234)
6	1.50	0.0558	0.3097	0.2539	0.9248	-0.0184	0.9248(Y-0.0198)
7	1.75	0.0497	0.269	0.2193	0.8621	-0.0149	0.8621(Y-0.0172)
8	2.00	0.0292	0.2631	0.2339	0.7849	-0.0109	0.7849(Y-0.0138)

4.1 Validation through graphical procedure

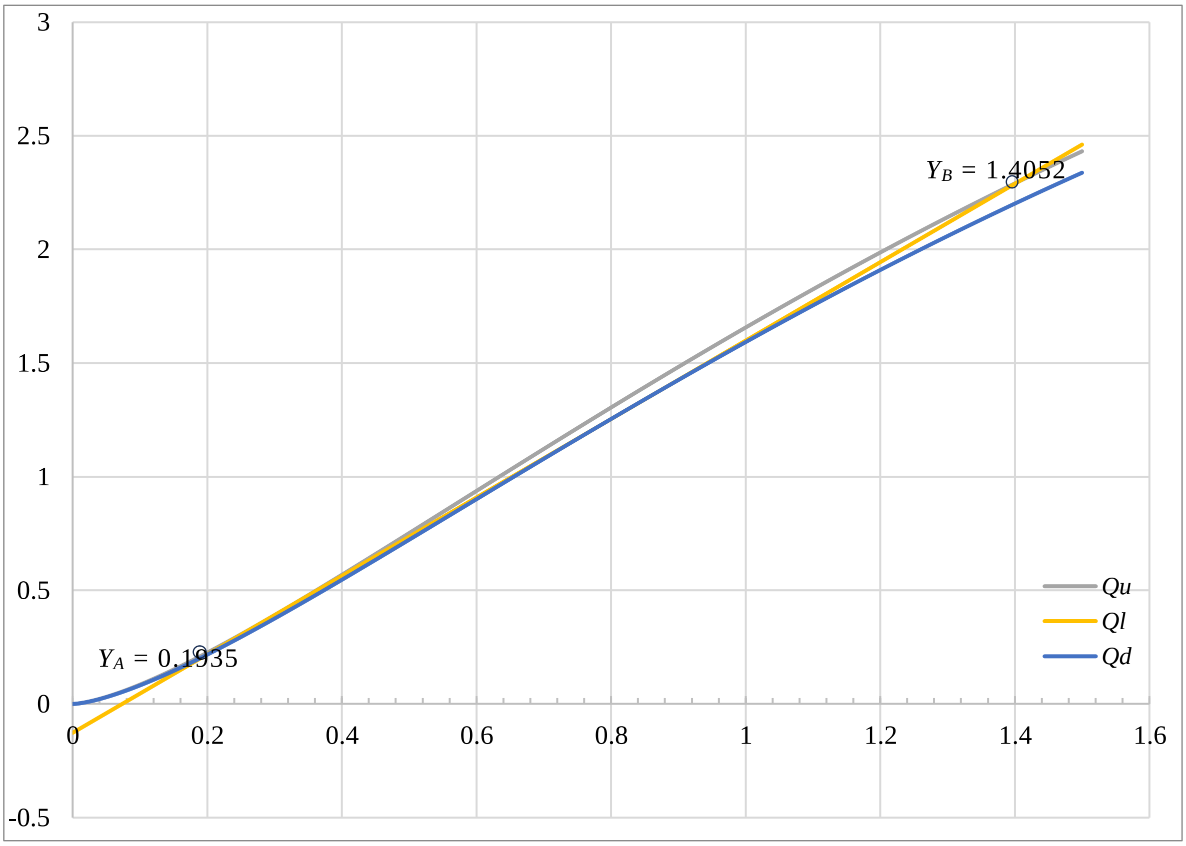


Fig 5: Depth-Discharge curves for flow thorough the Inward Trapezoidal Channel with z = 0.25

For a typical value of z, as can be seen from the graph (Fig. 5) that, the theoretical values obtained from the optimization procedure when plotted is as per the obtained values and hence validated.

5. Experiments

The experiments were carried out in the Hydraulics laboratory. The channel setup is as shown in plate 1. The experiments are conducted in open channels which are made of Perspex-glass, to minimize the friction effect. The test section of the channel is 1.2 x 0.075 x 0.15 m. The channel's straight length complies with IS norms for the approach length necessary to achieve a uniform flow. To get a uniform free-surface slope, wave dampener, in the form of thick cloth tied at the surface near the overhead tank is used. Experimental setup consists of an over-head tank of capacity 1000 litres (at the upstream) and measuring or collecting tank (on the downstream side) and the rectangular channel. The experiments were conducted for two different side slopes, z_1 = 0.095 and z_2 = 0.123 as shown in plate 2. To minimize the error and achieve accuracy, head readings are recorded with the Digital Vernier Scale with 0.01 mm least count along with auto detection of water level in the piezometer as shown in plate 3 and readings are taken in three locations upstream, intermediate stream and downstream of the channel to get the free-surface slope. The flow in the channel is collected in the collecting tank of size 0.5 x 0.35 x 0.5 m.

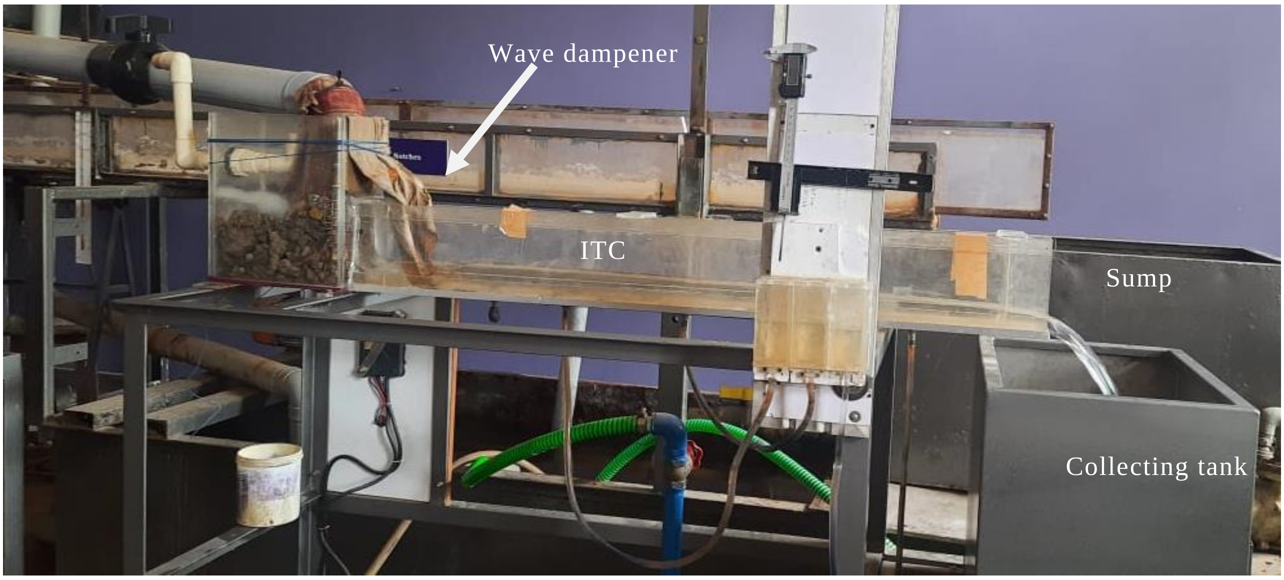
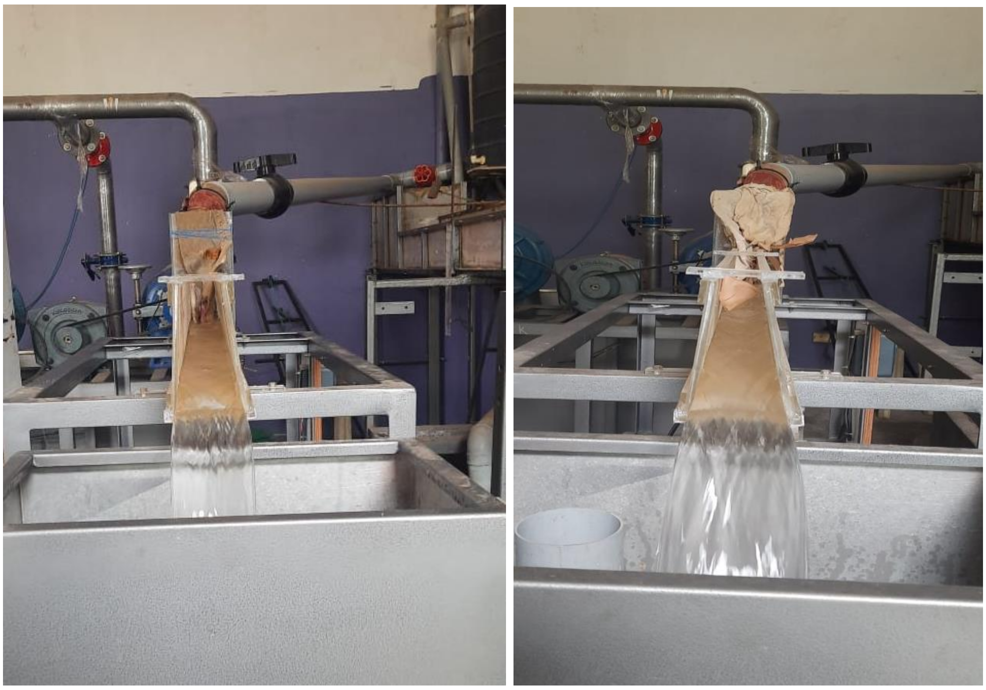


Plate 1: Experimental Setup



(a) **(b)**

Plate 2: Inward Trapezoidal Channel **(a)** z_1= 0.095 **and (b)** z_2 = 0.123

Plate 3: Digital Vernier Scale used for Depth measurement

6. Results and Discussion

It can be observed from inward trapezoidal channel, the proportionality between the flow depth and discharge is an increasing curve and there exist a near linear relationship between flow depth and discharge. The actual experimental values co-insides with the theoretical values as shown in Fig's 6 and 7. The actual and theoretical discharge equation is as follows $Q_{La} = 2.1725y - 0.3159$, $Q_{Lt} = 2.0171y - 0.1937$ for z_1=0.095 and $Q_{La} = 2.02y - 0.2027$, $Q_{Lt} = 2.0171y - 0.1937$ for $z_2 = 0.123$ within a maximum error deviation of ± 2 percent. The datum is the reference line from which the flow depth is reckoned from the depth of flow recorded from the channel bed.

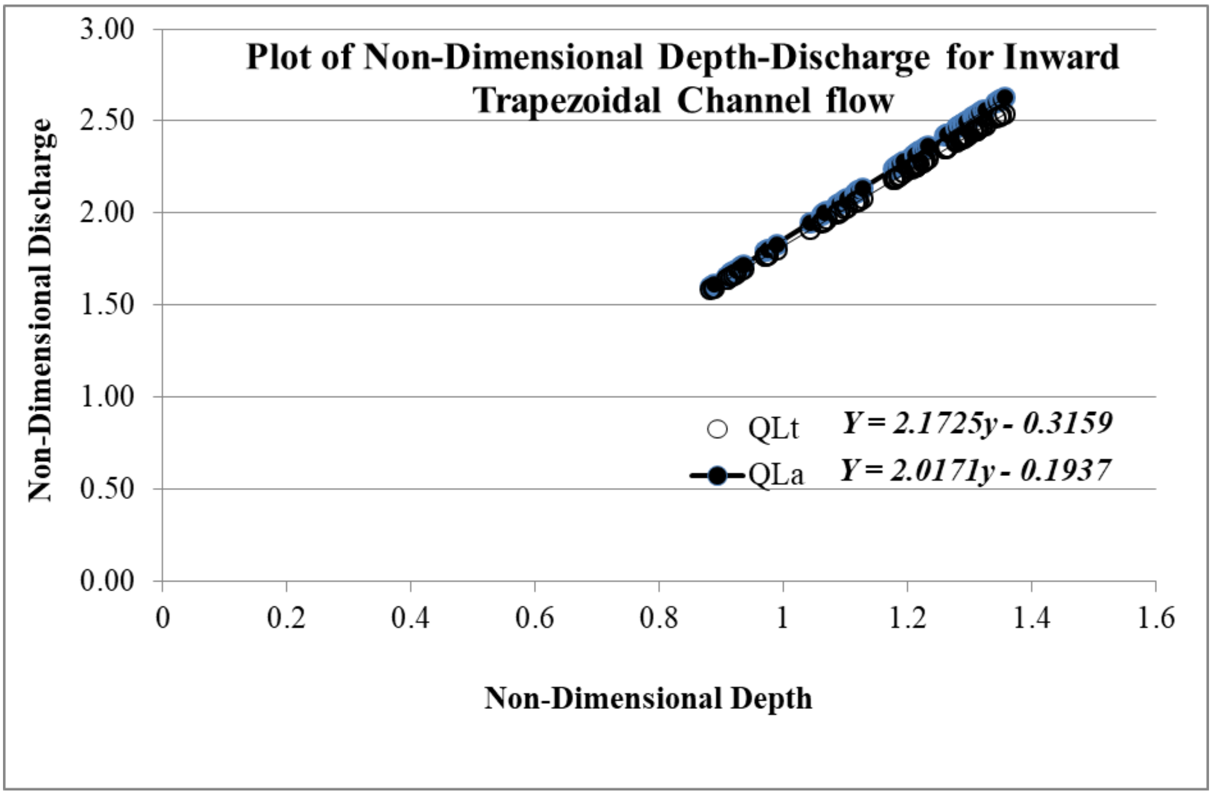


Fig 6: Actual and Theoretical Depth-Discharge Plot for z_1 = 0.095

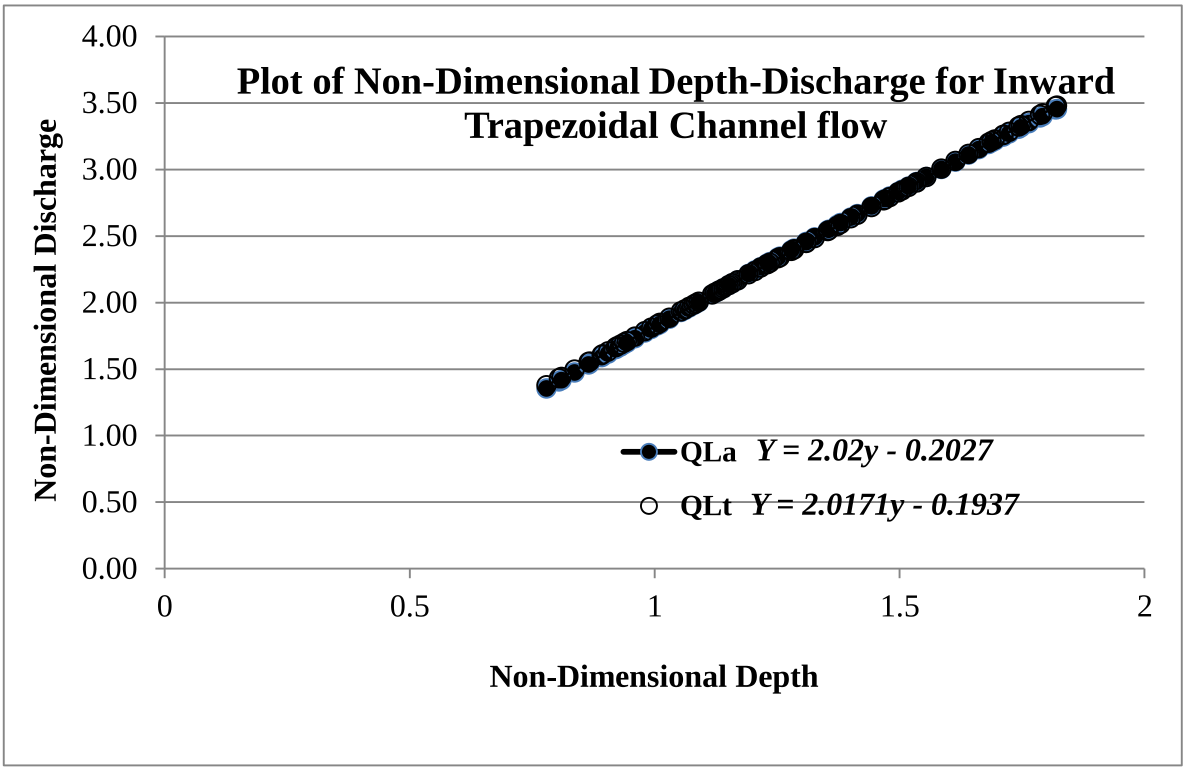


Fig 7: Actual and Theoretical Depth-Discharge Plot for z_2 = 0.123

5. Conclusions

The following conclusions are drawn from the analysis carried out for flow through ITC is as below:

- For the first time a theoretical and Experimental investigation on inward trapezoidal channel is carried out and algorithmic procedure has been developed to find the flow characteristics.
- The proposed linear discharge-depth relationship in inward trapezoidal channel (geometrically simple device), with least flow interference and near accurate measurement can be best alternative to the existing complicated measuring devices, either by construction or by calculations. In addition, no computations are required for measurement of discharges as it can be directly read and recorded on a scale which motivates even illiterate farmers to use such devices for effective cultivation and optimum usage of scarce resource, water. The simple linear discharge-depth equation that is proposed deviates less than 2% with the theoretical discharge. In addition, the threshold depth, beyond which the proposed linear depth-discharge relationship is valid, can be suitably designed as per the requirements. The linear relationship can be valid for either known or prefixed threshold depth or by knowing the height of the channel, the threshold depth can be fixed.
- The proposed linear equation is given by

Q_{La} = 2.1725y - 0.3159, Q_{Lt} = 2.0171y - 0.1937 for z_1=0.095 and

Q_{La} = 2.02y - 0.2027, Q_{Lt} = 2.0171y - 0.1937 for z_2=0.123

within a maximum deviation of ± 2 percent error.

- The proposed simple linear depth- discharge equation deviates less than 2% of computed discharge. Further, the near-linear depth-discharge relationship of inward trapezoidal channel with various values of z is presented.
- Further, as the discharge is linearly varying with the depth of flow, the discharges for various depths could be computed and the converted values of discharge printed on piezometer attached to the channel. In addition, the least count of the measurement decreases, increasing the sensitivity.
- In addition, a float-regulated device can be connected to a digital meter and the flat and discharge can be directly recorded in real-time similar to rain measuring gauges.
- The proposed inward trapezoidal flume is found to be highly useful in practice in Irrigation, Chemical, Hydraulic and Environmental engineering, by providing least interference in flow.

- The proposed ITC, shows better performance as the measurable range is concerned compared to rectangular channel flows.

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References

[1] Adarsh S and Shesha Prakash M N "Geometrically Simple Linear Channels" GIS Science Journal, ISSN No : 1869-9391 Volume 10, Issue 1, 2023, pp-358-365

[2] Chow, V.T., Open-channel Hydraulics, McGraw Hill, New York, 1959.

[3] E.Eyo and A.Offiong "Mathematical Modelling and its Impact on Open Channel Flow" Global Journal of Mathematical Sciences Volume 6, No 1, 2007, pp 9-15.

[4] Henderson, F.M. Open-Channel Flow McMillan, New York, 1967.

[5] Ismail Aydina, A. Burcu Altan-Sakarya, Cigdem Sisman 2011. "Discharge formula for rectangular sharp-crested weirs" Flow Measurement and Instrumentation 22 (2011) 144–151.

[6] Keshava Murthy, Alternate Rectangular Weir – Proportional Three Halves Power Weir, Journal of Institution of Engineers, India (Civil Engg. Divn) 1988, Vol. 68, pt. C15, pp. 231.

[7] Keshava Murthy and Giridhar D P, Inverted V-notch: A Proportional Weir, Jl. of Irrigation and Drainage Engineering Div. American Society for Civil Engineers, 1989, 115(6) Paper no, 1035-1050.

[8] Keshava Murthy and Shesha Prakash, An Alternate Design of a Square Rooting Measuring device in an Open Channel, Jl. of Irrigation and Drainage Engineering Div. American Society for Civil Engineers, Nov./Dec. 1995, 121(6) Paper no. 9519, 374-377. 2.

[9] Keshava Murthy and Shesha Prakash, Geometrically simple Logarithmic Weir, Jl. of Irrigation and Drainage Engineering Div. American Society for Civil Engineers, Nov./Dec. 1995, 121(6) Paper no. 9945, 419-426.

[10] Mohamad Reza Madadi, Ali Hosseinzadeh Dalir, Davood Farsadizadeh 2013. "Control of undular weir flow by changing of weir geometry" Flow Measurement and Instrumentation 34 (2013) 160–167.

[11] Mohamad Reza Madadi, Ali Hosseinzadeh Dalir, Davood Farsadizadeh 2014. “Investigation of flow characteristics above trapezoidal broad - crested weirs” Flow Measurement and Instrumentation 38 (2014) 139–148.

[12] Noor Aliza Ahmad, Saiful Iswal Saiful Bahry, Zarina Md Ali, Azra Munirah Mat Daud, and Sabariah Musa 2017. “Effect of Flow Resistance in Open Rectangular Channel” MATEC Web of Conferences 97, 01107 (2017) DOI: 10.1051/matecconf/20179701107 ETIC 2016.

[13] Prat, E.A., 1914, “Another Proportional Weir: Sutro Weir,” Engineering News Record, 72(9): 462-463.

[14] Seyed Vahid Nabavi 2015. “Flow Measurement in Rectangular Channels”, Bull. Env. Pharmacol. Life Sci., Vol 4 [Spl issue 1] 2015: 457-467.

[15] Shesha Prakash M N and Adarsh S, “Mathematical Modeling for Fluid Flow in Open Rectangular Channel”, Journal of Survey in Fisheries Sciences, Vol.10, Special Issue 4, April (2023), pp 3069-3075.

[16] Shesha Prakash M N and Adarsh S "Modelling for fluid flow with Linear Characteristics in Rectangular Open-Channel", International Journal of Scientific Research in Modern Science and Technology (IJSRMST), ISSN: 2583-7605 (Online), Volume 2, Issue 6, pp. 75-86, June 2023

[17] Subramanya, K., Tata McGraw - Hill Publishing Company Ltd., New Delhi, 1986.