

THE χ^2 OF TENSOR PRODUCTS IN DOUBLE ORLICZ SEQUENCE SPACES

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Abstract: Let X be a Banach lattice and χ_M^2 be an double gai Orlicz sequence space associated to an Orlicz function with the Δ_2 – condition. In this paper discuss the some general topological properties of tensor products.

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1 Introduction

Throughout w, χ, Γ and Λ denote the classes of all, gai, entire and analytic scalar valued single sequences, respectively.

We write w^2 for the set of all complex sequences (x_{mn}) , where $m, n \in \mathbb{N}$, the set of positive integers. Let (x_{mn}) be a double sequence of real or complex numbers. Then the series $\sum_{m,n=1}^{\infty} x_{mn}$ is called a double series. The double series $\sum_{m,n=1}^{\infty} x_{mn}$ is said to be convergent if and only if the double sequence (S_{mn}) is convergent, where

$$S_{mn} = \sum_{i,j=1}^{m,n} x_{ij} (m, n = 1, 2, 3, \dots) .$$

We denote w^2 as the class of all complex double sequences (x_{mn}) . A sequence $x = (x_{mn})$ is said to be double analytic if

$$\sup_{m,n} |x_{mn}|^{1/m+n} < \infty.$$

The vector space of all prime sense double analytic sequences are usually denoted by Λ^2 . A sequence $x = (x_{mn})$ is called double entire sequence if

$$|x_{mn}|^{1/m+n} \rightarrow 0 \text{ as } m, n \rightarrow \infty.$$

The vector space of all prime sense double entire sequences are usually denoted by Γ^2 . The space Λ^2 and Γ^2 is a metric space with the metric

$$d(x, y) = \sup_{m,n} \{|x_{mn} - y_{mn}|^{1/m+n}; m, n: 1, 2, 3, \dots\}, \quad (1)$$

for all $x = \{x_{mn}\}$ and $y = \{y_{mn}\}$ in Γ^2 . Let $\phi = \{\text{finite sequences}\}$.

A sequence $x = (x_{mn})$ is called double gai sequence if $((m+n)! |x_{mn}|)^{1/m+n} \rightarrow 0$ as

$m, n \rightarrow \infty$. The double gai sequences will be denoted by χ^2 .

Let M and Φ be mutually complementary Orlicz functions. Then, we have

(i) For all $u, y \geq 0$,

$$uy \leq M(u) + \Phi(y), \text{ (Young's inequality) [see, Kamphthan et al., (1981)]} \quad (2)$$

(ii) For all $u \geq 0$,

$$u\eta(u) = M(u) + \Phi(\eta(u)). \quad (3)$$

(iii) For all $u \geq 0$, and $0 < \lambda < 1$,

$$M(\lambda u) \leq \lambda M(u). \quad (4)$$

A sequence $M = (M_{mn})$ of Orlicz function is called a Musielak-Orlicz function. A sequence $g = (g_{mn})$ defined by

$$g_{mn}(v) = \sup\{|v|u - M_{mn}(u) : u \geq 0\}, m, n = 1, 2, \dots$$

is called the complementary function of a Musielak-Orlicz function M . For a given Musielak Orlicz function M , the Musielak-Orlicz sequence space t_M is defined by

$$M_f = \{x \in w^2 : I_M(|x_{mn}|)^{1/m+n} \rightarrow 0 \text{ as } m, n \rightarrow \infty\},$$

where I_M is a convex modular defined by

$$I_M(x) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} M_{mn}(|x_{mn}|)^{1/m+n}, x = (x_{mn}) \in t_M.$$

We consider t_M equipped with the Luxemburg metric

$$d(x, y) = \sup_{m, n} \left\{ \inf \left(\sum_{m=1}^{\infty} \sum_{n=1}^{\infty} M_{mn} \left(\frac{|x_{mn}|^{1/m+n}}{mn} \right) \right) \leq 1 \right\}.$$

The postivity perspective, it is know that the projective tensor and the injective tensor product of two Banach lattices are, in general not Banach lattices.

Throughout w, χ and Λ denote the classes of all, gai and analytic scalar valued single sequences, respectively. We write w^2 for the set of all complex double sequences (x_{mn}) , where $m, n \in \mathbb{N}$, the set of positive integers. Then, w^2 is a linear space under the coordinate wise addition and scalar multiplication.

2 Notations

For a vector space X , a vector $\bar{x} = (x_{ij})_{i,j} \in X^{\mathbb{N} \times \mathbb{N}}$ and $n \in \mathbb{N}$, we write $\bar{x}(\leq n)$ is a two dimensional matrix from first term to nth term and remaining term zero. and $\bar{x}(> n)$ is a two dimensional matrix from first term to nth term zero and start with (n+1)th term.

If X is an ordered set, the usual order on $X^{\mathbb{N} \times \mathbb{N}}$ is defined by $\bar{x} = (x_{ij})_{i,j} \geq 0 \Leftrightarrow x_{ij} \geq 0$ for each $i, j \in \mathbb{N}$. for Banach lattice X , X^* denotes its dual space, B_X denotes its closed unit ball, and X^+ denotes its positive cone.

3 Definitions and Preliminaries

A sequence $x = (x_{mn})$ is said to be double analytic if

$\sup_{m,n} |x_{mn}|^{\frac{1}{m+n}} < \infty$. The vector space of all double analytic sequences is usually denoted by

Λ^2 . A sequence $x = (x_{mn})$ is called double entire sequence if $|x_{mn}|^{\frac{1}{m+n}} \rightarrow 0$ as $m, n \rightarrow \infty$.

The vector space of double entire sequences is usually denoted by Γ^2 . A sequence $x = (x_{mn})$

is called double gai sequence if $((m+n)! |x_{mn}|)^{\frac{1}{m+n}} \rightarrow 0$ as $m, n \rightarrow \infty$. The vector space

of double gai sequences is usually denoted by χ^2 . The space χ^2 is a metric space with the metric

$$d(x, y) = \sup_{m,n} \left\{ ((m+n)! |x_{mn} - y_{mn}|)^{\frac{1}{m+n}} : m, n, k: 1, 2, 3, \dots \right\} \quad (5)$$

for all $x = \{x_{mn}\}$ and $y = \{y_{mn}\}$ in χ^2 .

Let $n \in \mathbb{N}$ and X be a real vector space of dimension w , where $n \leq m$. A real valued function $d_p(x_1, \dots, x_n) = \| (d_1(x_1, 0), \dots, d_n(x_n, 0)) \|_p$ on X satisfying the following four conditions:

(i) $\| (d_1(x_1, 0), \dots, d_n(x_n, 0)) \|_p = 0$ if and only if $d_1(x_1, 0), \dots, d_n(x_n, 0)$ are linearly dependent,

(ii) $\| (d_1(x_1, 0), \dots, d_n(x_n, 0)) \|_p$ is invariant under permutation,

(iii) $\| (\alpha d_1(x_1, 0), \dots, d_n(x_n, 0)) \|_p = |\alpha| \| (d_1(x_1, 0), \dots, d_n(x_n, 0)) \|_p, \alpha \in \mathbb{R}$

(iv) $d_p((x_1, y_1), (x_2, y_2), \dots, (x_n, y_n)) = (d_X(x_1, x_2, \dots, x_n)^p + d_Y(y_1, y_2, \dots, y_n)^p)^{1/p}$ for $1 \leq p < \infty$; (or)

(v) $d((x_1, y_1), (x_2, y_2), \dots, (x_n, y_n)) := \sup\{d_X(x_1, x_2, \dots, x_n), d_Y(y_1, y_2, \dots, y_n)\}$,

for $x_1, x_2, \dots, x_n \in X, y_1, y_2, \dots, y_n \in Y$ is called the p product metric of the Cartesian product of n metric spaces is the p norm of the n -vector of the norms of the n subspaces.

A trivial example of p product metric of n metric space is the p norm space is $X = \mathbb{R}$ equipped with the following Euclidean metric in the product space is the p norm:

$$\| (d_1(x_1, 0), \dots, d_n(x_n, 0)) \|_E = \sup(|\det(d_{mn}(x_{mn}, 0))|) =$$

$$\sup \left(\left| \begin{array}{cccc} d_{11}(x_{11}, 0) & d_{12}(x_{12}, 0) & \dots & d_{1n}(x_{1n}, 0) \\ d_{21}(x_{21}, 0) & d_{22}(x_{22}, 0) & \dots & d_{2n}(x_{2n}, 0) \\ \vdots & \vdots & \ddots & \vdots \\ d_{n1}(x_{n1}, 0) & d_{n2}(x_{n2}, 0) & \dots & d_{nn}(x_{nn}, 0) \end{array} \right| \right)$$

where $x_i = (x_{i1}, \dots, x_{in}) \in \mathbb{R}^n$ for each $i = 1, 2, \dots, n$.

If every Cauchy sequence in X converges to some $L \in X$, then X is said to be complete with respect to the p – metric. Any complete p – metric space is said to be p – Banach metric space.

3.1 Definition

Positive tensor products: For Banach lattices X and Y , let $X \otimes Y$ denote the algebraic tensor product of X and Y . For each

$$u = \sum_{m=1}^r \sum_{n=1}^s x_{mn} \otimes y_{mn} \in X \otimes Y,$$

define $T_u: X^* \rightarrow Y$ by $T_u(x^*) = \sum_{m=1}^r \sum_{n=1}^s x^*(x_{mn})y_{mn}$ for each $x^* \in X^+$.

Then injective cone on $X \otimes Y$ is defined to be $C_i = \{u \in X \otimes Y: T_u(x^*) \in Y^* \quad \forall x^* \in X^{**}\}$.

3.2 Definition

Let $X \overline{\otimes}_i Y$ denote the completion of $X \otimes Y$ with respect $d(\cdot, \cdot)$. Then $X \overline{\otimes}_i Y$ with C_i as its positive cone is a Banach lattice called the positive injective tensor product of X and Y .

The positive cone on $x \otimes Y$ is defined to be

$$C_p = \{\sum_{m=1}^r \sum_{n=1}^s x_{mn} \otimes y_{mn}: r, s \in \mathbb{N}, x_{mn} \in X^+, y_{mn} \in Y^+\}.$$

We define the following spaces:

For a Banach metric lattice X , let

$$\chi_M^2(X) = \left\{ \bar{x} = (x_{ij})_{ij} \in X^{\mathbb{N} \times \mathbb{N}}: \left(x^* \left((i+j)! |x_{ij}| \right)^{1/i+j} \right)_{ij} \in \chi^2, \forall x^* \in X^{**} \right\}.$$

The metric defined to be

$$d(x, y) = \sup \left\{ \left\| \left(x^* \left((i+j)! |x_{ij} - y_{ij}| \right)^{1/i+j} \right) : x^* \in B_{X^{**}} \right\| \right\}, \quad x = (x_{ij})_{ij} \in \chi_M^2(X).$$

$$\text{Let } \chi_M^2(X) = \left\{ \bar{x} \in \chi_M^3(X): \lim_{i,j,n} \left\| \left((i+j)! |\bar{x}_{ij}(>n)| \right)^{1/i+j} \right\| \rightarrow 0 \text{ as } i, j \rightarrow \infty \right\},$$

$$\text{with the metric } d(x, y) = \sup \left\{ \left\| \left((i+j)! |(\bar{x}_{ij} - \bar{y}_{ij})(>n)| \right)^{1/i+j} \right\| \forall (\bar{x}_{ij}) \in \chi_M^2(X^{**}) \right\}.$$

4 Some New Orlicz sequence spaces of Tensor product

The main aim of this article is to introduce the following sequence spaces and examine the topological and algebraic properties of the resulting sequence spaces. Let $M = (M_{mn})$ be a sequence Musielak-Orlicz functions,

$$\left(\bar{X}, \left\| (d(\bar{x}_1, 0), d(\bar{x}_2, 0), \dots, d(\bar{x}_{n-1}, 0)) \right\|_p \right) \text{ be a } p\text{- metric space, and consider } \mu_{mn}(\bar{x}) =$$

$$\left\| \left((i+j)! |\bar{x}_{ij}(>n)| \right)^{1/i+j} \right\| \text{ and } \eta_{mn}(\bar{x}) = \left\| |\bar{x}_{ij}(>n)|^{1/i+j} \right\|.$$

We define the following sequence spaces as follows:

$$\left[\chi_M^2, \left\| (d(\bar{x}_1, 0), d(\bar{x}_2, 0), \dots, d(\bar{x}_{n-1}, 0)) \right\|_p \right] =$$

$$\lim_{mn \rightarrow \infty} \left\{ \sum_m \sum_n \left[M_{mn} \left(\left\| \mu_{mn}(\bar{x}), (d(\bar{x}_1, 0), d(\bar{x}_2, 0), \dots, d(\bar{x}_{n-1}, 0)) \right\|_p \right) \right] \right\},$$

and

$$\left[\Lambda_M^2, \left\| (d(\bar{x}_1, 0), d(\bar{x}_2, 0), \dots, d(\bar{x}_{n-1}, 0)) \right\|_p \right]$$

$$= \sup \left\{ \sum_m \sum_n \left[M_{mn} \left(\left\| \eta_{mn}(\bar{x}), (d(\bar{x}_1, 0), d(\bar{x}_2, 0), \dots, d(\bar{x}_{n-1}, 0)) \right\|_p \right) \right] \right\}$$

$$< \infty \left. \right\}.$$

5 Main Results

5.1 Theorem

Let $M = (M_{mn})$ be a sequence Musielak-Orlicz functions, the tensor product of Orlicz sequence spaces $\left[\chi_{M\mu}^2, \left\| (d(x_1), d(x_2), \dots, d(x_{n-1})) \right\|_p \right]$ and $\left[\Lambda_{M\eta}^2, \left\| (d(x_1), d(x_2), \dots, d(x_{n-1})) \right\|_p \right]$ are linear spaces.

Proof: It is routine verification. Therefore the proof is omitted.

5.2 Theorem

Let $M = (M_{mn})$ be a sequence Musielak-Orlicz functions, the tensor product of Orlicz sequence space $\left[\chi_{M\mu}^2, \left\| (d(x_1), d(x_2), \dots, d(x_{n-1})) \right\|_p \right]$ is a paranormed space with respect to the paranorm defined by

$$g(x) = \left\{ \left[M_{mn} \left(\left\| \mu_{mn}(x), (d(x_1), d(x_2), \dots, d(x_{n-1})) \right\|_p \right) \right] \right\}.$$

Proof: Clearly $g(x) \geq 0$ for $x = (x_{mn}) \in \left[\chi_{M\mu}^2, \left\| (d(x_1), d(x_2), \dots, d(x_{n-1})) \right\|_p \right]$. Since $M_{mn}(0) = 0$, we get $g(0) = 0$.

Conversely, suppose that $g(x) = 0$, then

$$\left\{ \left[M_{mn} \left(\left\| \mu_{mn}(x), (d(x_1), d(x_2), \dots, d(x_{n-1})) \right\|_p \right) \right] \right\}$$

Suppose that $\mu_{mn}(x) \neq 0$ for each $m, n \in \mathbb{N} \times \mathbb{N}$. Then $\left\| \mu_{mn}(x), (d(x_1), d(x_2), \dots, d(x_{n-1})) \right\|_p \rightarrow \infty$. It follows that

$\left(\left[M_{mn} \left(\left\| \mu_{mn}(x), (d(x_1), d(x_2), \dots, d(x_{n-1})) \right\|_p \right) \right] \right) \rightarrow \infty$ which is a contradiction. Therefore

$\mu_{mn}(x) = 0$. Let

$$\left(\left[M_{mn} \left(\left\| \mu_{mn}(x), (d(x_1), d(x_2), \dots, d(x_{n-1})) \right\|_p \right) \right] \right)$$

and

$$\left(\left[M_{mn} \left(\left\| \mu_{mn}(y), (d(x_1), d(x_2), \dots, d(x_{n-1})) \right\|_p \right) \right] \right).$$

Then by using Minkowski's inequality, we have

$$\begin{aligned} & \left(\left[M_{mn} \left(\left\| \mu_{mn}(x+y), (d(x_1), d(x_2), \dots, d(x_{n-1})) \right\|_p \right) \right] \right) \leq \\ & \left(\left[M_{mn} \left(\left\| \mu_{mn}(x), (d(x_1), d(x_2), \dots, d(x_{n-1})) \right\|_p \right) \right] \right) + \\ & \left(\left[M_{mn} \left(\left\| \mu_{mn}(y), (d(x_1), d(x_2), \dots, d(x_{n-1})) \right\|_p \right) \right] \right). \end{aligned}$$

So we have

$$\begin{aligned} g(x+y) &= \left\{ \left[M_{mn} \left(\left\| \mu_{mn}(x+y), (d(x_1), d(x_2), \dots, d(x_{n-1})) \right\|_p \right) \right] \right\} \\ &\leq \left\{ \left[M_{mn} \left(\left\| \mu_{mn}(x), (d(x_1), d(x_2), \dots, d(x_{n-1})) \right\|_p \right) \right] \right\} \\ &+ \left\{ \left[M_{mn} \left(\left\| \mu_{mn}(y), (d(x_1), d(x_2), \dots, d(x_{n-1})) \right\|_p \right) \right] \right\} \end{aligned}$$

Therefore,

$$g(x+y) \leq g(x) + g(y).$$

Finally, to prove that the scalar multiplication is continuous. Let λ be any complex number.

Therefore paranormed by,

$$g(\lambda x) = \left\{ \left[M_{mn} \left(\left\| \mu_{mn}(\lambda x), (d(x_1), d(x_2), \dots, d(x_{n-1})) \right\|_p \right) \right] \right\}. \text{ Hence it is continuous.}$$

This completes the proof.

5.3 Theorem

(i) If the sequences of Musielak-Orlicz functions (M_{mn}) and (N_{mn}) are satisfies Δ_2 – condition, then

$$\begin{aligned} & \left[\chi_{M\mu}^2, \left\| \mu_{mn}(x), (d(x_1), d(x_2), \dots, d(x_{n-1})) \right\|_p \right] = \\ & \left[\chi_{N\mu}^2, \left\| \mu_{uv}(x), (d(x_1), d(x_2), \dots, d(x_{n-1})) \right\|_p \right]. \end{aligned}$$

(ii) If the sequences of Musielak-Orlicz functions (M_{mn}) and (N_{mn}) are satisfies Δ_2 – condition, then

$$\begin{aligned} & \left[\chi_{N\mu}^2, \left\| \mu_{mn}(x), (d(x_1), d(x_2), \dots, d(x_{n-1})) \right\|_p \right] = \\ & \left[\chi_{M\mu}^2, \left\| \mu_{mn}(x), (d(x_1), d(x_2), \dots, d(x_{n-1})) \right\|_p \right] \end{aligned}$$

Proof: Let the sequences of Musielak-Orlicz functions (M_{mn}) and (N_{mn}) are satisfies Δ_2 – condition, we get

$$\left[\chi_{N\mu}^2, \left\| \mu_{mn}(x), (d(x_1), d(x_2), \dots, d(x_{n-1})) \right\|_p \right] \subset$$

$$\left[\chi_{N\mu}^2, \|\mu_{mn}(x), (d(x_1), d(x_2), \dots, d(x_{n-1}))\|_p \right] \quad (6)$$

To prove the inclusion

$$\left[\chi_{M\mu}^2, \|\mu_{mn}(x), (d(x_1), d(x_2), \dots, d(x_{n-1}))\|_p \right] \subset \left[\chi_{N\mu}^2, \|\mu_{mn}(x), (d(x_1), d(x_2), \dots, d(x_{n-1}))\|_p \right],$$

let $a \in \left[\chi_{M\mu}^2, \|\mu_{mn}(x), (d(x_1), d(x_2), \dots, d(x_{n-1}))\|_p \right]$. Then for all $\{x_{mn}\}$ with $(x_{mn}) \in$

$$\left[\chi_{M\mu}^2, \|\mu_{mn}(x), (d(x_1), d(x_2), \dots, d(x_{n-1}))\|_p \right] \text{ we have}$$

$$\sum_{m=1}^{\infty} \sum_{n=1}^{\infty} |x_{mn} a_{mn}| < \infty. \quad (7)$$

Since the Musielak-Orlicz functions sequence (M_{mn}) satisfies Δ_2 - condition, then

$$(y_{mn}) \in \left[\chi_{M\mu}^2, \|\mu_{mn}(x), (d(x_1), d(x_2), \dots, d(x_{n-1}))\|_p \right], \text{ we get } \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \left| \frac{y_{mn} a_{mn}}{(m+n)!} \right| < \infty.$$

by (7). Thus $(a_{mn}) \in \left[\chi_{M\mu}^2, \|\mu_{mn}(x), (d(x_1), d(x_2), \dots, d(x_{n-1}))\|_p \right] =$

$$\left[\chi_{N\mu}^2, \|\mu_{mn}(x), (d(x_1), d(x_2), \dots, d(x_{n-1}))\|_p \right] \text{ and hence}$$

$(a_{mn}) \in \left[\chi_{N\mu}^2, \|\mu_{mn}(x), (d(x_1), d(x_2), \dots, d(x_{n-1}))\|_p \right]$. This gives that

$$\left[\chi_{M\mu}^2, \|\mu_{mn}(x), (d(x_1), d(x_2), \dots, d(x_{n-1}))\|_p \right] \subset$$

$$\left[\chi_{N\mu}^2, \|\mu_{mn}(x), (d(x_1), d(x_2), \dots, d(x_{n-1}))\|_p \right] \quad (8)$$

we are granted with (7) and (8)

$$\left[\chi_{M\mu}^2, \|\mu_{mn}(x), (d(x_1), d(x_2), \dots, d(x_{n-1}))\|_p \right] =$$

$$\left[\chi_{N\mu}^2, \|\mu_{mn}(x), (d(x_1), d(x_2), \dots, d(x_{n-1}))\|_p \right]$$

(ii) Similarly, one can prove that $\left[\chi_{N\mu}^2, \|\mu_{mn}(x), (d(x_1), d(x_2), \dots, d(x_{n-1}))\|_p \right] \subset$

$\left[\chi_{M\mu}^2, \|\mu_{mn}(x), (d(x_1), d(x_2), \dots, d(x_{n-1}))\|_p \right]$ if the sequence (N_{mn}) satisfies Δ_2 - condition.

5.4 Proposition

The tensor product of Orlicz sequence space $\left[\chi_{M\mu}^2, \|\mu_{mn}(x), (d(x_1), d(x_2), \dots, d(x_{n-1}))\|_p \right]$

is not solid.

Proof: The result follows from the following example.

Example: Consider

$$x = (x_{mn}) = \begin{pmatrix} 1 & 1 & \dots & 1 \\ 1 & 1 & \dots & 1 \\ \vdots & & & \\ \vdots & & & \\ 1 & 1 & \dots & 1 \end{pmatrix} \in \left[\chi_{M\mu}^2, \|\mu_{mn}(x), (d(x_1), d(x_2), \dots, d(x_{n-1}))\|_p \right].$$

Let

$$\alpha_{mn} = \begin{pmatrix} (-1)^{m+n} & (-1)^{m+n} & \dots & (-1)^{m+n} \\ (-1)^{m+n} & (-1)^{m+n} & \dots & (-1)^{m+n} \\ \vdots & & & \\ \vdots & & & \\ (-1)^{m+n} & (-1)^{m+n} & \dots & (-1)^{m+n} \end{pmatrix}, \text{ for all } m, n \in \mathbb{N} \times \mathbb{N}.$$

Then $\alpha_{mn}x_{mn} \notin \left[\chi_{M\mu}^2, \|\mu_{mn}(x), (d(x_1), d(x_2), \dots, d(x_{n-1}))\|_p \right]$. Hence

$\left[\chi_{M\mu}^2, \|\mu_{mn}(x), (d(x_1), d(x_2), \dots, d(x_{n-1}))\|_p \right]$ is not solid.

5.5 Proposition

The tensor product of Orlicz sequence space $\left[\chi_{M\mu}^2, \|\mu_{mn}(x), (d(x_1), d(x_2), \dots, d(x_{n-1}))\|_p \right]$ is not monotone.

Proof: The proof follows from Proposition 5.4.

5.6 Theroem

The tensor product of Orlicz sequence spaces $\left[\chi_{\mu}^2, \|(d(x_1), d(x_2), \dots, d(x_{n-1}))\|_p \right]$ and $\left[\Lambda_{\eta}^2, \|(d(x_1), d(x_2), \dots, d(x_{n-1}))\|_p \right]$ are not convergence free.

Example: Consider, $\left[M_{mn} \left(\|\mu_{mn}(x), (d(x_1), d(x_2), \dots, d(x_{n-1}))\|_p \right) \right] = \left[M \left(\|\mu_{mn}(x), (d(x_1), d(x_2), \dots, d(x_{n-1}))\|_p \right) \right] = \left[\left(\|\mu_{mn}(x), (d(x_1), d(x_2), \dots, d(x_{n-1}))\|_p \right) \right]$ for all $m, n \in \mathbb{N} \times \mathbb{N}$, for m, n are odd, If m, n even, consider the sequence $(x_{mn}) = (mn)^{-(m+n)}$ for all $m, n \in \mathbb{N} \times \mathbb{N}$ belongs to each of $\left[\chi_{\mu}^2, \|(d(x_1), d(x_2), \dots, d(x_{n-1}))\|_p \right]$. and $\left[\Lambda_{\eta}^2, \|(d(x_1), d(x_2), \dots, d(x_{n-1}))\|_p \right]$. Consider the sequence (y_{mn}) defined by $(y_{mn})^{1/m+n} = m^2n^2$, for all $m, n \in \mathbb{N} \times \mathbb{N}$. Then (y_{mn}) neither belongs to $\left[\chi_{\mu}^2, \|(d(x_1), d(x_2), \dots, d(x_{n-1}))\|_p \right]$ nor $\left[\Lambda_{\eta}^2, \|(d(x_1), d(x_2), \dots, d(x_{n-1}))\|_p \right]$. Hence the tensor product Orlicz sequence spaces are not convergence free.

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