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A MATHEMATICAL MODEL OF AERODYNAMIC FORCES IN AVIAN FLIGHT

Nasiha Saher Bano and S.V. Suneetha
Department of Mathematics, Rayalaseema University, Kurnool - 518002, India
E-mail: nasihaifthikar@yahoo.com, rsu2072@yahoo.com

Abstract: A simple mathematical model is proposed for the calculation of aerodynamic forces like lift, thrust, drag and gravitational force (weight) knowing basic body parameters of birds understudy. The study suggests that all aerodynamic forces are governed by the angle of attack. The wing beat frequency mostly remains to be constant, whatever may be the style and mode of the flight. The data on lift, drag, thrust, coefficient of lift and coefficient of drag are presented and discussed critically.
Keywords: Mathematical model, Lift, Drag, Thrust, Weight, Avian flight.

1. Introduction

In natural fliers, the production of aerodynamic forces (lift and thrust) are mainly due to the complex movement of wings and associated flight muscles of a flier, while in airplanes and helicopters, there exist different devices to generate and control aerodynamic forces.

There will be considerable resistance offered by all bodies, when they move through fluid media. The study of lift and drag is essential. The function of lift is to overcome the body weight and the drag is resistance offered to oppose the motion of the body. The frictional drag of wings caused by viscous forces in air, represents loss of power.

Pennycuick [1] described a method for measuring the effective lift: drag ratio of a bird in flapping flight knowing the bird's span ratio i.e., the ratio of the wing span in the up stroke to that in the down stroke. For this method, it is assumed that the circulation of the wing is constant and the lift is proportional to the wing span. He concluded that the method can be applied in the field to double- crested carmorants in their steady continuous flapping flight. Adeel Ahmad, et. al., [2] calculated the aerodynamic forces-lift and drag and their coefficients of the smallest insect, mosquito by developing a theory by considering the wings of a flier as harmonic oscillator. Here, the lift of mosquito was measured by developing a simple technique using digital single pan electrical balance. The calculated value of lift was verified with the experimental. Their study throws light on morpho- physiological adaptation of mosquitoes for the generation of aerodynamic forces in hovering, tethered and forward flights. Syed Riaz Ahmed, et. al. [3] calculated the lift and drag coefficients of a honey bee

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(*Apisdorsata*) in the state of hovering by knowing the basic body parameters. The calculated value of lift was verified with the experimental value. The study throws light on morpho-physiological adaptation of *Apisdorsata* for the generation of aerodynamic forces in steady state aerodynamics. Philips et. al., [4] presented a lifting line theory for a flapping wings in steady forward flight in which the unsteady features of the flow were modeled. A detailed three-dimensional model of the vortex wake was used to evaluate the unsteadiness to the first order. The method gave satisfactory agreement with well-known limiting cases. Relationship between the geometric and kinematic parameters; and the forces and power were predicted which were compatible with the experimental evidence.

A search of literature reveals that extensive work on aerodynamic forces in insect flight has been done, but it is not much in the case of bird flight. Hence, an attempt is made to build a simple mathematical model for the computation of lift and thrust in avian flight.

2. A proposed Model for the Aerodynamic Forces acting on a flier in its forward flapping Flight

A simple model is developed for a flier (insect or bird or bat) flying in the state of forward flapping flight. During the flight the forces acting on the flier are lift, gravitational force (weight), thrust and drag.

A flier can be air borne when the upward force, the lift is balanced by the gravitational force, the weight. It moves in the forward direction when thrust overcomes the opposing force, the drag which is generated by the motion of the body in air, due to the wing beat and also wing blown in the opposite direction to that of the flier. A free body diagram of forces acting on a flier is shown in Fig. 1.

In the model it is assumed that the force responsible for the forward flight or hovering flight is the reacting force which is generated by the air pushed in the downward direction due to wing beat. The components of the reacting force (R), the lift (L) and thrust (T) are conditioned by the angle of attack (ϕ) of the flier. The angle of attack can be defined as the angle made by the body axis of the flier with the horizontal. The wing plane angle γ is the angle between body axis and wing plane axis as shown in Fig.1. The reacting force (R) acts always perpendicular to the wing plane. Let the reacting force (R) makes an angle θ with the vertical axis.

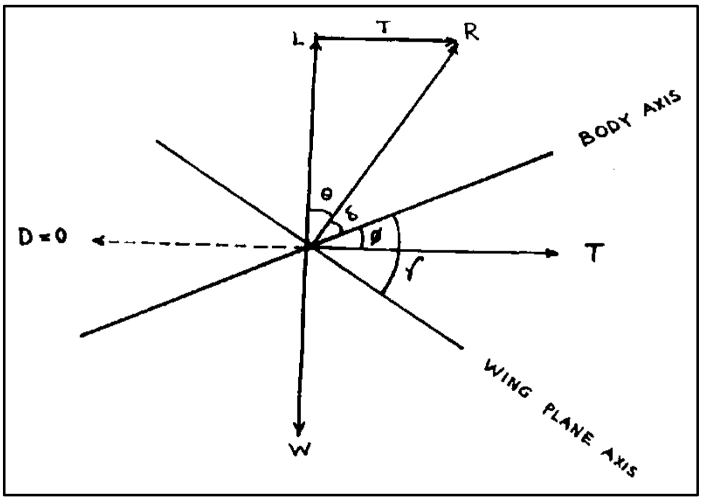

Fig. 1. Free body diagram aerodynamic forces of bird in flight

From the free body diagram

$L = R \cos\theta$ (1)

$T = R \sin \theta - D$ (2)

But $\theta + \delta + \phi = 90°$ (3)

$\gamma + \delta = 90°$ (4)

From equations (3) and (4.),

$\theta - \gamma + \phi = 0$

In general, $\theta = \gamma - \phi$ (5)

for insects, $\gamma = 60°$

$\theta = 60° - \phi$ (6)

for birds and bats

$\gamma = 90°$

$\theta = 90° - \phi$ (7)

Considering the rate of change of momentum of air induced in the down ward direction, the reacting force in terms of V_i is written as

$R = 2 S_d V_i^2 \rho$

$W = L = R \cos\theta = (2 S_d V_i^2 \rho) \cos(r - \phi)$ (8)

$T = R \sin \theta - D = 2 S_d V_i^2 \sin(r - \phi) - D$ (9)

Since $D << R$

$T = 2 S_d V_i^2 \rho \sin(r - \phi)$ (10)

$L = (2 S_d V_i^2 \rho) \cos(\gamma - \phi)$ (11)

(1). For hovering flight

L = W; T = 0; D = 0

(2). For forward flapping flight

L = W; T >> D

In order to examine the effect of angle of attack (ϕ) on lift (L) and thrust (T), L and T have been calculated by changing ϕ in the interval of 10°, and the data is tabulated in tables.

3. Materials and Methods

The birds selected for the present investigation are 25 species. They were purchased mostly from birds dealer, Hyderabad. Some were collected from Pakhallake and Bhadrakali tank, Warangal. After making measurements birds were set free to fly. The body mass (M_f) of the birds was measured using common digital balance with an accuracy of 1 gm. The wing length (l), wing span (L_w) and wing area (A) were measured by stretching the wings properly and pinned to a drawing board. The contour of the wing was drawn on drawing sheet carefully along the wing margins. A meter scale was used to measure wing length and wing span, while planimeter was employed to measure wing area. The wing beat frequency of some birds were observed, when they were hovering in nature. In the case of small birds with relatively high frequency, the wing beat frequency was measured under stroboscopic flash using electronic stroboscope.

4. Results and Discussion

Table 1 gives data on lift acting on birds in forward flight at different angles of attack ranging from 10^0 to 80^0. Lift increases non-linearly with the increase in the angle of attack. When the angle becomes 90^0, lift is maximum and horizontal flight changes to hovering flight. Similarly, Table 2 shows data on thrust at different angles of attack in the range of 10^0 to 80^0 with an interval of 10^0. Thrust decreases non-linearly as angle of attack increases.

Table 1. Data on Lift acting on birds in forward flight at different angle of attack

Flier	R (dyne)	Lift, L (dyne) at angle (degree) of attack of							
		10	20	30	40	50	60	70	80
Passeriformes									
Nactoriniaasiatica	5841	1011	1998	2920	3756	4474	5058	5490	5753
Estrildaamandava	8448	1462	2889	4224	5432	6471	7316	7941	8321
Loncuramalabarica	10035	1737	3432	5018	6453	7687	8690	9433	9885
Loncurepunculata	11858	2053	4055	5929	7625	9083	10269	11147	11680
Loncuramalacca	15268	2643	5221	7634	9818	11696	13222	14352	15039
Passer domesticus	21756	3766	7441	10878	13989	16665	18841	20451	21430
Dicrurusadsimilis	40376	6989	13809	20188	25962	30928	34966	37953	39770

Turdoidesstriatus	47432	8210	16222	23716	30499	36333	41076	44586	46721
Acridotherustristis	75656	13096	25874	37828	48647	57952	65518	71117	74521
Corvussplendens	244804	42374	83723	122402	157409	187520	212000	230116	241131
Psittaciformes									
Melosittacusundulatus	25676	4444	8781	12838	16510	19668	22235	24135	25290
Psittaculacathorpae	46648	8075	15954	23324	29995	35732	40397	43849	45948
Psittaculakrameri	99960	17303	34186	49980	64274	76569	86565	93962	98461
Pelecaniformes									
Egerettagarzetta	246372	42646	84259	123186	158418	188721	213358	231590	242676
Ardeolagravii	265972	46038	90962	132986	171020	203735	230332	250014	261982
Bubulaus ibis	272244	47124	93107	136122	175053	208539	235763	255909	268160
Coraciformes									
Meropsorientalis	14896	2578	5094	7448	9578	11410	12900	14002	14673
Upupaapops	34104	5903	11664	17052	21929	26124	29534	32058	33592
Coraciasindica	95060	16454	32511	47530	61124	72816	82322	89356	93634
Apodiformes									
Apusaffinus	16542	2863	5658	8271	10637	12671	14326	15550	16294
Galliformes									
Peridiculaastatica	33908	5869	11597	16954	21803	25974	29364	31874	33399
Strigiformes									
Bubo bengalensis	122304	21170	41828	61152	78641	93685	105915	114966	120469
Falconiformes									
Falco peregrinus	134456	23274	45984	67228	86455	102993	116439	126389	132439
Gruiformes									
Grusgrus	142688	24699	48799	71344	91748	109299	123568	134127	140548
Columbiformes									
Columba livia	198156	34300	67769	99078	127414	151788	171603	186267	195184

R: Reacting Force

Table 2. Data on Thrust acting on birds in forward flight at different angle of attack

Flier	R (dyne)	Thrust, T (dyne) at angle (degree) of attack of							
		10	20	30	40	50	60	70	80
Passeriformes									
Nactoriniaasiatica	5841	5753	5490	5058	4474	3756	2920	1998	1016
Estrildaamandava	8448	8320	7941	7316	6471	5432	4224	2889	1470
Loncuramalabarica	10035	9884	9433	8690	7687	6453	5018	3432	1746
Loncurepunculata	11858	11679	11147	10269	9083	7625	5929	4055	2063
Loncuramalacca	15268	15038	14352	13222	11696	9818	7634.	5222	2657
Passer domesticus	21756	21428	20451	18841	16665	13989	10878	7441	3786
Dicrurusadsimilis	40376	39767	37953	34966	30928	25962	20188	13809	7025
Turdoidesstriatus	47432	46716	44586	41076	36333	30499	23716	16222	8253
Acridotherustristis	75656	74514	71117	65518	57952	48647	37828	25874	13164
Corvussplendens	244804	241109	230116	212000	187520	157409	122402	83723	42596
Psittaciformes									
Melosittacusundulatus	25676	25289	24135	22235	19668	16510	12838	8781	4468
Psittaculacathorpae	46648	45944	43849	40397	35732	29995	23324	15954	81167
Psittaculakrameri	99960	98451	93962	86565	76569	64274	49980	34186	17393
Pelecaniformes									
Egerettagarzetta	246372	242653	231590	213358	188721	158417	123186	84259	42869.
Ardeolagravii	265972	261957	250014	230332	203735	171020	132986	90962	46279
Bubulaus ibis	272244	268134	255909	235763	208539	175053	136122	93107	47370

Coraciformes									
Meropsorientalis	14896	14671	14002	12900	11410	9578	7448	5094	2592
Upupaapops	34104	33589	32058	29534	26124	21929	17052	11664	5934
Coraciasindica	95060	93625	89356	82322	72816	61124	47530	32511	16540
Apodiformes									
Apusaffinus	16542	16293	15550	14326	12671	10637	8271	5658	2878
Galliformes									
Peridiculaastatica	33908	33396	31874	29364	25974	21803	16954	11597	5900
Strigiformes									
Bubo bengalensis	122304	120458	114966	105915	93685	78641	61152	418278	21281
Falconiformes									
Falco peregrinus	134456	132426	126389	116439	102993	86455	67228	45984	23395
Gruiformes									
Grusgrus	142688	140534	134127	123568	109299	91748	71344	48800	24828
Columbiformes									
Columba livia	198156	195165	1862667	171603	151788	127414	99078	67770	34479

R: Reacting Force

The wing profile of bird is varied at each point along its length from fulcrum to the tip of the wing. The curvature and thickness is more at the base of the wing, but it decreases gradually towards the wing tip. When air-stream flows over the surface of an airfoil, the unsymmetrical stream lines generate pressure difference around the airfoil. The air stream flows more vigorously over the dorsal convex surface of the airfoil, than over the resisting ventral surface and it creates less pressure or otherwise a negative pressure. Therefore, the velocity of air will be more at the upper surface. Hence, it creates a greater dynamic pressure (kinetic pressure) per unit volume of fluid, With the result of this pressure, the atmospheric pressure (static pressure) above the surface of the wings gets decreased. This results in lifting the airfoil.

The angle of attack is the angle between the plane of wing chord and the direction of the airflow. In the present study, for the horizontal flight, angle of attack is defined as the angle the flier makes with horizontal. The higher the angle of attack, the greater is the lift generated. The various other pressures generated along with the angle of attack also create the lift.

In the down stroke, the wing twists towards the tip to make the angle of attack gradually smaller. Consequently, the lift generated by the wing changes continuously from base to tip. The proximal part of the wing, consisting of secondary and tertiary feathers, being near to the body moves over a relatively short distance during the wing beat cycle. The lift produced by this part of the wing, being perpendicular to the nearby horizontal airstream, is directed upwards and bears most of the weight of the bird. At the distal part of the wing, consisting of primary feathers, i.e., at the wing tip which is moving at a higher speed, the airstream is generated mainly by the beating of the wing. The lift so produced is directed more forward because of the forward motion during the down stroke and thus provides thrust. During the up stroke the proximal part of the wing produces less lift while no thrust is generated by the wing. Since large birds beat their wings relatively slow, they keep themselves airborne by generating lift in both the upstroke and down stroke. On the other hand, small birds find it difficult to move forward and hence must generate as much thrust as possible during the down stroke as well as in the up stroke. Thus, it is interesting to note that lift is the dominant force in large birds while drag is the dominant force in small birds. Small and relatively medium sized birds can practice hovering, i.e., stationary flight, flying at one spot without any forward motion.

The present study concludes:

A constant reacting force is produced by the mass flow of air induced and, vertical and horizontal components of which are lift and thrust respectively. These essential forces (lift and thrust) are controlled and regulated by the angle of attack of the flier.

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