

## **SOME CONCEPTIONS NEED TO BE MODIFIED IN THE REFERENCES ON THEORY OF RELATIVITY**

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**Abstract:** This article shows that the erroneous conception: “mass is relative, its value depends on velocity” continue to be used in current references on the theory of relativity of some countries. This article points out the unreasonableness when using the concept of relativistic mass and suggests questions that need to be modified.

**Keywords:** Mass; energy; momentum; theory of relativity; Einstein's formula.

### **1. INTRODUCTION**

Formulated by Albert Einstein in 1905, relativity is one of the most famous scientific theories of the 20th century. The special theory of relativity was based on postulates that the speed of light in vacuum was independent of the motion of all observers and the laws of physics are the same for all non-accelerating observers. Following this logic, statements about space and time, distances and duration turn out to be relative. Einstein found that space and time were interwoven into a single continuum known as space-time. He then spent ten more years trying to include acceleration in the theory and finally publishing his theory of general relativity. In this addition to special relativity, he found that massive objects cause a distortion in space-time which is felt as gravity. Einstein's theory of relativity is the basis for our understanding of the universe. The content of this theory is now written in many languages in the world. However, there are two different schools on relativistic dynamics: one embraces speed-dependent “relativistic mass” and the other rejects it, maintaining that there is only one mass and it's speed-independent. This article points out the unreasonableness when using the concept of “relativistic mass”. This is not a novel result; nevertheless, many continue to use this concept and some have even attempted to establish it as the basis for special relativity. This article also finds out the reasons for the erroneous conclusion about the dependence of mass on velocity and suggests questions that need to be modified.

## 2. MASS IN THE REFERENCES ON THEORY OF RELATIVITY

The concept of mass has always been fundamental to physics. The mass of an object is a numerical measure of its inertia; a fundamental measure of the amount of matter in the object. Mass in special relativity incorporates the general understandings from the laws of motion of special relativity along with its concept of mass-energy equivalence. In the paper written in 1905 “Does the Inertia of a Body Depend upon its Energy-Content” [1], Einstein used  $L$  to denote energy and arrived at a conclusion which may be written as the equation :

$$m = L/c^2 \quad (1)$$

where  $c$  is the speed of light. This expression of the equivalence of mass and energy entered the science so firmly that it became a symbol of the theory of relativity and a criterion of its practical significance. Now if we use the more modern symbol instead of  $L$  to denote energy, how this equation may be rewritten? There are two schools on this issue, therefor the word mass is given two meanings, one equivalent rest energy  $E_0$  :

$$m = E_0/c^2 \quad (2)$$

the other equivalent total energy  $E$  :

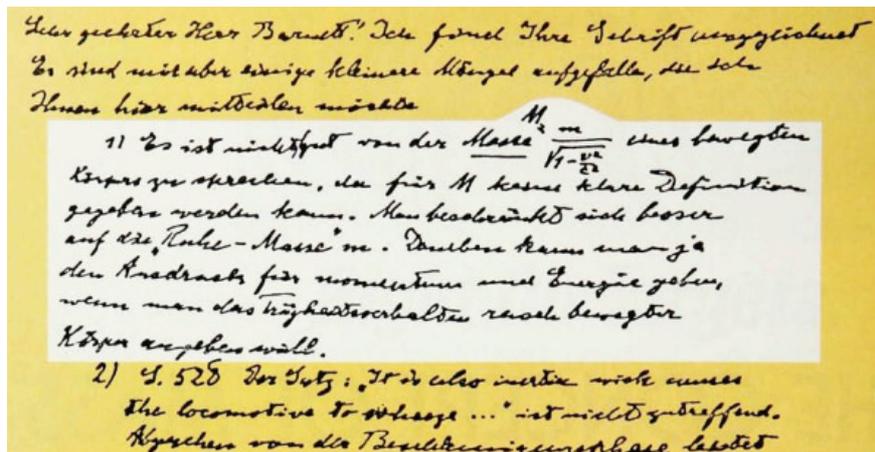
$$m = E/c^2 \quad (3)$$

If using the equation (2) then mass is an invariant quantity which is the same for all observers in all reference frames. If using the equation (3) then mass is dependent on the velocity, the body at rest has a “rest mass”  $m_0$ , whereas a body moving with velocity  $v$  has “relativistic mass” or “mass”  $m$ , given by:

$$m = m_0 / \sqrt{1 - v^2/c^2} \quad (4)$$

It's easy to find dozens of physics books that discuss the relativistic mass as equation (4). By placing the derivation of  $m(v)$  in a section called something like “Einstein's Theory of Relativity”, most of these books imply that Einstein was responsible for equation (4) - that's not true! Einstein never approved of relativistic mass [2]. His first paper on relativity appeared when the concept of a speed-dependent electromagnetic mass had already become a topic of considerable interest. First, he accepted this idea and wrote the formula for the longitudinal  $m_\ell$  and transverse mass  $m_t$  [3]. But after Planck introduced relativistic momentum in 1906 [4], Einstein abandoned speed-dependent mass. From then on mass was invariant. When the term relativistic mass became popular, Einstein silently eschewed it. By then he was well into the general theory and for him  $m$  was “an invariant tensor of rank zero”

[5]. Later in life, he addressed the issue directly in a letter to L. Barnett (figure 1) [6]: “It is not good to introduce the concept of the mass  $M = m/\sqrt{1-v^2/c^2}$  of a moving body for which no clear definition can be given. It is better to introduce no other mass concept than the 'rest mass'  $m$ . Instead of introducing  $M$  it is better to mention the expression for the momentum and energy of a body in motion”.



**Figure 1.** Letter from Albert Einstein to Lincoln Barnett, 19 June 1948.

Einstein wrote in German [6].

In 1987 C. Adler published the paper “Does mass really depend on velocity, dad?” in the American Journal of Physics [7]. The question posed in the title was put to the author by his son. The answer was: “No!” “Well, yes...”, “Actually, no, but don't tell your teacher”. The next day his son dropped physics. The interpretation of special relativity that the mass of a relativistic body increases with velocity is an unfortunate consequence of relativity. “Time does not wait. Every year books are published in millions of copies that hammer into the heads of the young generations false ideas about the theory of relativity. This process must be stopped.” - these words were said by L.B. Okun [8], one of the most persistent and consistent adherents of the position that there is only one mass, and it is speed-independent. There have been many articles written to address this issue, but it is obvious that even today equation (4) can be found on different language editions of the free encyclopedia Wikipedia (Bengal, Hebrew, Vietnam, Greek, Sec, ...) [9-13]. Many media, including textbooks and popular writings of some countries, continue to use the term “relativistic mass” to mean an increase in the measured mass when an entity is moving at relativistic speeds. In his commentary on how this issue has been treated in the media and in textbooks, Okun wrote: “Unfortunately, sometimes and especially in his popular writings Einstein was careless about the subscript 0 and spoke about the equivalence of mass and energy and omitted the attribute “rest” for the

energy. As a result Einstein's equation  $E_0 = mc^2$  became known in its famous but misleading form  $E = mc^2$  ... "To substantiate the formula  $m = E/c^2$  some authors use the connection between momentum and velocity in Newtonian mechanics,  $\vec{p} = m\vec{v}$ , forgetting that this relation is valid only when  $v \ll c$ "... [14].

### 3. WHY IS RELATIVISTIC MASS CONSIDERED A BAD CONCEPT?

The term "relativistic mass" can be easily seen to be inconsistent. For example, one cannot observe changes in the mass of an object as a function of the speed of an observer relative to the object. So, if a fast rocket passes near you and someone in there looks at you, you will not increase your mass. In other words, you will not become a black-hole if you move fast enough. The association of "relativistic mass" with gravitational effects is fundamentally incorrect; for example the gravitational attraction between the photon and a large mass (the Sun) is determined by their energy-momentum tensors, not just by their energies [15]. Basically, the effect relativistic mass is trying to explain ... but it is not a particularly meaningful definition. Exactly how is it becoming more massive? Are there more atoms in the object? How does one actually measure this increase? Can one actually measure it? The effects of time dilation and length contraction can be explicitly measured in simple ways. Any change in mass can not be measured directly but only inferred. In addition, the use of relativistic mass is based on heuristic grounds and is not a fundamental part of the theory. Why employ an unnecessary concept that is furthering confusion? [16].

The foremost reason that relativistic mass is introduced in works directed at the general public is that it is sufficient to explain why no material object can travel at or beyond the speed of light. According to this viewpoint the closer the object's speed is to  $c$ , the greater the increase in mass, so to reach the speed of light would require an infinitely strong force acting on the body. The problem with this explanation is that  $m = m_0 / \sqrt{1 - v^2/c^2}$  usually cannot be considered as inertia (in theory of relativity the coordinate acceleration for a given force depends on direction [7]), furthermore, for it does not imply the addition of velocity formula and therefore two observers in two inertial frames of reference moving in opposite directions may observe the other to be traveling at a speed greater than  $c$ . Relativistic mass in itself is not sufficient to give a speed limit for any object, yet if it is introduced along with the principle of special relativity it is superfluous [16]. The constancy of the speed of light is sufficient to give a fundamental speed limit.

The main motivation behind introduction of relativistic mass in more formal texts is maintaining familiar, Newtonian-like, expressions for velocity and momentum. These derivations posit a Newtonian form for the 3-momentum:

$$\vec{p} = m\vec{v} \quad (5)$$

and then impose conservation of momentum, provided that the classical concept of mass is modified. According to the idea of four dimensional space-time introduced by Minkowski the positions of a particle in space  $\vec{r}$  and in time  $t$  form a 4-vector  $x^\mu = (ct, x, y, z)$ . By asserting (5) as the definition of relativistic momentum, one is forced to adopt a primitive concept of an improper 4-velocity; for  $v^\mu = dx^\mu/dt$  is not Lorentz covariant. Lorentz transforming from a frame where an object is moving with a 3-velocity  $\vec{v}$  to its rest frame that  $v'^\mu = (c/\gamma, 0, 0, 0)$ , whereas in its rest frame it is required to be  $(c, 0, 0, 0)$ . The improper velocity being a direct result of the imposition of relativistic mass means that relativistic mass is at odds with the accepted kinematics of special relativity [16].

Naturally the result found is that the new relativistic expression for momentum, that is Lorentz covariant, must be of the form:

$$\vec{p} = m\vec{v}/\sqrt{1-v^2/c^2} \quad (6)$$

and total energy  $E$  of a body in motion:

$$E = mc^2/\sqrt{1-v^2/c^2} \quad (7)$$

Here  $m$  the ordinary mass, the same as in Newtonian mechanics. The relation between  $E$  and  $\vec{p}$  is:

$$E^2 = m^2c^4 + p^2c^2 \quad (8)$$

Such a mass is defined by the relation:

$$m^2 = E^2/c^4 - p^2/c^2 \quad (9)$$

Like the 4-dimensional coordinates  $t$  and  $\vec{r}$ , the energy  $E$  and the momentum  $\vec{p}$  are the components of a 4-dimensional vector  $p^\mu (E/c, p_x, p_y, p_z)$ . In order to have a Lorentz covariant 4-momentum, the mass is found to be a Lorentz invariant, which means that  $m$  has the same value in all reference frames, and hence, does not depend on velocity.

Here if we use the term relativistic mass and rest mass then it will lead to the form of 4-momentum  $(mc, p_x, p_y, p_z)$ . In the classic relativity textbook Spacetime Physics [17] Taylor and Wheeler write: "Ouch! The concept of 'relativistic mass' is subject to misunderstanding.

That's why we don't use it. First, it applies the name mass --belonging to the magnitude of a 4-vector -- to a very different concept, the time component of a 4-vector. Second, it makes increase of energy of an object with velocity or momentum appear to be connected with some change in internal structure of the object. In reality, the increase of energy with velocity originates not in the object but in the geometric properties of space-time itself'.

Back to misleading form of Einstein's relation  $E = mc^2$ , according to this formula the mass of the body increases with increasing velocity of the body and the photon has a mass:

$$m = E/c^2 = h/\lambda c \quad (10)$$

where  $\lambda$  is the wave length of light,  $h$  is the Planck constant. Formula (10) shows that the relativistic mass of photons is frequency dependent, UV photons are more energetic than visible photons, and so are more "massive" in this sense!!! Agree with this viewpoint the "rest mass" of photons is zero. Whether it makes sense to talk about the "rest mass" of a particle that can never be at rest??? The answer, again, is that "rest mass" is really a misnomer, and it is not necessary for a particle to be at rest for the concept of mass to make sense. Technically, it is the invariant length of the particle's four-momentum. For all photons this is zero [18]. Formula (9) shows that the relation between momentum and energy for a photon is:

$$p = E/c \quad (11)$$

On the other hand, from (6) and (7) we obtain:

$$\vec{v} = c^2 \vec{p}/E \quad (12)$$

Formulas (11) and (12) combine together give the speed of photon  $v = c$ , this mean the photon always moves at the speed of light.

#### 4. CONCLUSIONS

Despite the general usage of invariant mass in the scientific literature, the use of "relativistic mass" is still found in many media. The experience of answering confused questions about relativistic mass suggests that its use is not helpful. The mass is an intrinsic property of a body, and it does not depend on the observer's frame of reference. Invariant mass proves to be more fundamental in four-dimensional space-time approach to special relativity, and relativistic mass is of no use at all in general relativity. This is a strong argument against teaching relativistic mass to students who will go on to more advanced levels.

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