

## FLOW MODELLING OF INCLINED TRAPEZOIDAL WEIR WITH A NEW APPROACH

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**Abstract:** Trapezoidal weir is the combination of rectangular and triangular weirs. The discharge through trapezoidal weir will be combination of the two rectangular and triangular weirs so that for a given head it will be relatively greater in both the weirs. Researchers have attempted to improve the discharge coefficient of these weirs with fair degree of accuracy. Although these weirs have relatively better degree of accuracy, they are avoided in field measurements due to its major drawback in development of afflux. Many researchers have attempted placing weirs inclined to the bed of the channel to reduce the afflux. This procedure of practice of the thin plate weirs of standard geometric shape has increased the discharging coefficient. This paper is concerned with the experimentation, analysis and development of discharge-head-inclination model to measure flow over inclined trapezoidal weir with a new approach. The new method is simple and the error analysis shows that the estimated discharge is well within 2%(except very few data points nearer to the crest and 60° and 75° weir inclination) and less than the previous methods of flow analysis. The research work is of considerable academic interest in showing that the head index is independent of weir inclination and depends on the shape of the weir opening. It is shown that the discharge coefficient increases with increase in weir angle with respect to normal (vertical) position, thereby reducing the afflux which may prompt field channel users to implement these weirs with simple head-discharge-inclination equation. The weir position of 75°with respect to vertical plane along the flow direction has been experimented and analysed. The Mathematical modeling results in a single head discharge inclination equation which can be used for any trapezoidal weir of any desired inclination.

**Keywords:** Thin plate weirs, Inclined weirs, Trapezoidal weirs, Discharge characteristics, Flow measurement, Afflux, Inclined-weir discharging index.

### Introduction

The combination of rectangular and triangular weirs is called trapezoidal weir. These weirs have the benefits of both the weirs and also overcome the individual limitations. The discharge through trapezoidal weir will be combination of the two rectangular and triangular weirs so that for a given head it will be relatively greater in both the weirs. Sharp crested weirs are relatively accurate flow measuring devices and are most commonly used devices in Open channels. Due to its major drawback in development of afflux and large head loss in the

form of coefficient of discharge, they are seldom used in the field channels. To reduce the afflux and to increase the discharging capacity, a number of investigations on weir profiles have been done by many researchers. Generally the weirs are fixed normal to the flow axis and lot of experimental work is available through the past literature. The literature in this paper is restricted to analysis of flow through the inclined weirs.

Henderson (1), Subramanya and Awasthy (2), Nadesamoorthy and Thomson (3), Ranga Raju et al (4), Hager (5), Singh, et al (6) have analysed flow over rectangular side weir.

Tullis et al (7) studied Labyrinth weir having trapezoidal plan forms and presented results in the form of curves between discharge coefficient and E/w with angle ' $\theta$ ' as the third parameter.

Ramamurthy et al. (8) have tested trapezoidal side weirs, Rahimpour et al. [9] investigated the discharge coefficient of sharp crested trapezoidal side weirs.

Hadis Haddadi, and Majid Rahimpour (9) investigated the flow over trapezoidal broad-crested side weir in rectangular channels under subcritical flow conditions and found that the discharge coefficient of a trapezoidal broad-crested side weir is related to the Froude number at the upstream of the weir, ratios of weir height to depth of flow, weir length to width of main channel and length of broad-crested weir to width of main channel for different side slopes.

Shesha Prakash and Shivapur (10, 11) have studied the variation of discharge coefficient with the angle of inclination of Trapezoidal weir plane with respect to the normal (standard) position of plane of weir for a sharp crested trapezoidal weir. Most of the researchers have reported their study on establishing discharge coefficient in terms of weir head to weir (height/length) and skew angle. The majority of these investigators have used other type of sharp crested weirs to calibrate the weir which is under investigation, limiting the accuracy of the results obtained.

Shesha Prakash et.al (12, 13) has worked on flow through inclined weirs with different openings like rectangular, triangular, trapezoidal and Inverted V-Notch (IVN). They have developed a mathematical model for the flow through weirs and a software to compute the head-discharge equation for the given type of weir and inclination.

In both the cases the results depended on the modeling and the both coefficients, viz., weir discharge coefficient and head index were developed on the basis of experimental values.

It is obvious that most of the investigators have studied the hydraulic characteristics of rectangular side weirs and less attention has been given to the behavior of flow over sharp-crested trapezoidal side weirs.

In the present investigation, a sharp-crested trapezoidal weir of 70 mm crest width and side slope of 1:4 is fixed inclined with respect to the bed of the channel and a different approach is followed to obtain the weir discharge coefficient for flow through inclined trapezoidal weir.

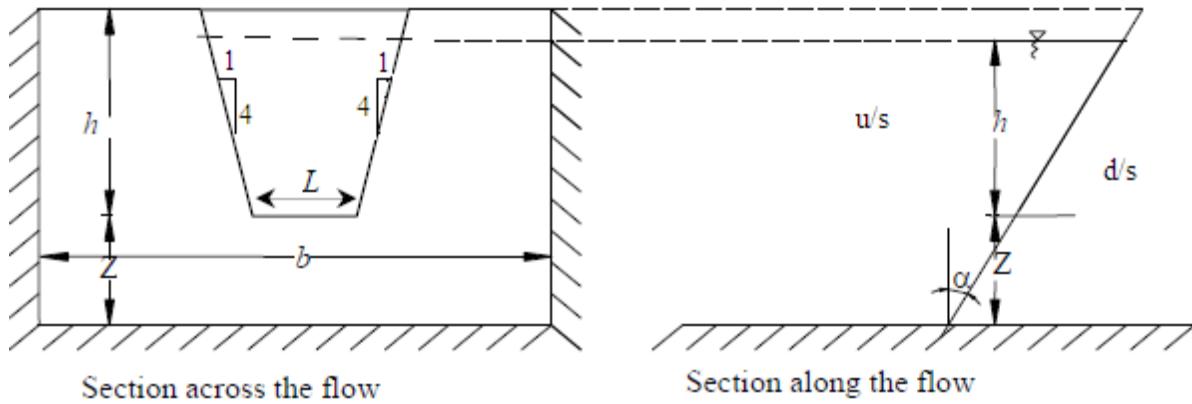


Fig 1. Arrangement of inclined Trapezoidal Weir

Fig.1 shows the sketch of inclined trapezoidal weir and can be seen that the flow length along the plane of the weir increases with increase in inclination with the normal plane which increases the discharging capacity of the weir.

#### Formulation of the Problem:

The discharge for flow through trapezoidal weir,  $\left(\frac{v^2}{2g}\right)$ , the velocity of approach is very small it is ignored) by considering the flow to be combination of rectangular and triangular weir, is given by Darcy-Weisbach equation as

$$q = \left(\frac{2}{3}\right) \sqrt{2g} w^{\frac{5}{2}} h^{\frac{3}{2}} + \left(\frac{8}{15}\right) \sqrt{2g} \tan \theta w^{\frac{5}{2}} h^{\frac{5}{2}} \quad (01)$$

Initially the obtained and computed values of Head and discharge were non-dimensionalised as below so that obtained equation will be more generic in nature.

Where  $q$  is the discharge through the weir in  $m^3/s$ ,  $2w$  is the crest width in m and  $h$  is the head over the crest in m.

Non-dimensionalising the above equation, we get

$$Q = \left[ \frac{2}{3} H^{\frac{3}{2}} + \frac{4}{15} \tan(\theta) H^{\frac{5}{2}} \right] \quad (02)$$

where  $Q = \frac{q}{2\sqrt{2g} w^{\frac{5}{2}}}$  and  $H = \frac{h}{w}$

### Experiments

Experiments were carried on inclined trapezoidal notch fixed normal to the flow direction ( $0^\circ$ ,  $15^\circ$ ,  $30^\circ$ ,  $45^\circ$ ,  $60^\circ$  and  $75^\circ$ ) inclinations with respect to the normal plane (Vertical) along the flow axis. The experimental channel is rectangular in section and having dimensions 0.3m wide, 0.2m deep and of 4.6m length. The channel is constructed of Perspex sheet and has smooth walls and nearly horizontal bed to reduce the boundary frictional force. It is connected to a Head tank of capacity 750 liters. The inclined trapezoidal weir is made of 8mm stainless steel with a crest thickness of 1 mm and a  $45^\circ$  chamfer given on downstream side to get a springing nappe. The experimental set up is shown in Fig. 2.



Fig.2 Experimental Setup

Water is supplied to the channel by an inlet valve provided on supply pipe. Overhead tank is provided with overflow arrangement to maintain constant head. Smooth, undisturbed, steady-uniform flow was obtained by making the water to flow through graded aggregates and the surface waves were damped by tying gunny bags at the surface near the tank. The head over the weir is measured using an electronic point gauge placed in piezometer located at a distance of about 1.40m on upstream of inclined trapezoidal notch. A collecting tank of size 1 m length, 0.6 m breadth, and of 0.6 m depth is provided with a piezometer. Water after running through the experimental setup is collected in a sump from which it is re-circulated by pump by lifting it back to the overhead tank.

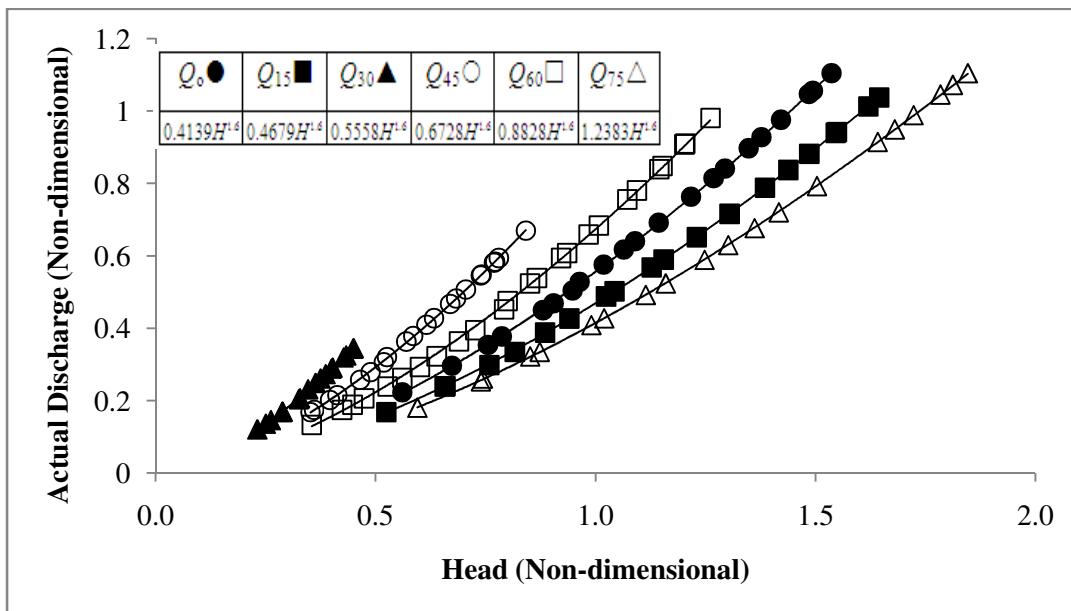
In the present study, the conventional method of volumetric discharge measurement is used, which increases the accuracy of the work. The measurements are done through electronic point gauge which automatically detects the water level and records the gauge reading. The volumetric measurement is done through self regulated timer for a fixed rise of water level automated through sensors. To eliminate the human error in piezometric readings in the collecting tank, time was auto recorded by an electronic timer, for the predefined interval of rise of water in the tank and averaged, by considering the cumulative volume and the accumulated time.

The present investigation was carried out on the range of variables shown in Table1

<b>Position of the weir</b>	<b>Normal</b>	<b>Angle of inclination with vertical plane in degrees</b>				
	0	15	30	45	60	75
Head over the crest (m)	20.85	18.38	19.64	12.43	12.34	8.10
	64.64	60.30	53.79	44.15	29.48	15.76
Actual Discharge	0.3665	0.3419	0.4520	0.2695	0.3412	0.2444
	2.2433	2.2687	2.2435	1.9915	1.3584	0.6990
No. of runs	38	34	30	26	19	21

### Analysis of results

A plot of Non-dimensional discharge verses non-dimensional head for various positions of plane of trapezoidal weir have been shown in Fig.3. It shows that the discharge increases with increase in inclination angle  $\alpha$ .



**Fig. 3:** Non-dimensional Head-discharge plot for various values of inclination  $\alpha$ .

Hence, sharp-crested trapezoidal weir can be installed at a suitable inclination in the channel without any alteration to the conventional simple geometry of the weir so that the discharging capacity of the weir can be much higher, corresponding to the same head, as compared to conventional normal weir, which is evident from Fig. 3. This will help in reducing free board requirement on upstream of weir position. Further, it can also be used in the existing channels with least effect of afflux.

### **Mathematical Modeling:**

Cowgill and Banks have shown that the head-discharge equation is a function of weir profile equation and proved that the profile equation for head-discharge equation of relationship  $Q \propto H^n$  will be given by  $y \propto x^{(n-\frac{1}{2})}$ . From Cowgill and Banks, for a rectangular weir with profile equation  $y \propto x$ , the head-discharge equation will be  $Q \propto H^{(1+\frac{1}{2})} \Rightarrow \phi H^{\frac{3}{2}}$ , where  $K$  is the discharge coefficient for the weir and angle of inclination. With this it can be seen that the Head index is a function of weir profile and is assumed to be nearly constant at 1.5. Hence the discharge coefficient will be a function of weir inclination  $\alpha$ , i.e.  $\phi = f(\alpha)$ .

The discharge-head-inclination equation can be expressed as

$$Q = f(\alpha) H^{1.5} \quad (03)$$

This method reduces the complicated two dimensional variation of weir discharging index and head index to simple weir discharging index.

A programmable algorithm is used to obtain a second order polynomial to get the inclined-weir-discharging index as follows:

The modeling part is subdivided into two stages.

Initially the Actual and theoretical discharge values for corresponding weir inclinations are tabulated as  $Q_{ai}$  and  $Q_{ti}$ . The corresponding weir discharging index for given inclination is found by the following equation:

$$\phi_i = \frac{\sum Q_{ai}^2}{\sum Q_{ai} Q_{ti}}$$

The modeling consists of directly getting the inclined weir discharging index by performing regression analysis directly on non-dimensional, actual and theoretical discharges for corresponding inclinations. Fig. 4 shows the variation Non-dimensional actual discharge with theoretical discharge for corresponding inclinations. As the theoretical discharge equation is derived for normal position of the weir, the coefficient will be obtained as an index of the

corresponding inclined head-discharge equation with respect to normal position, thereby giving a better weir discharging index ( $\phi$ ).

$$\phi_i = \frac{(Q_a)_i}{(Q_t)_n}$$

The head-discharge equation for flow through any sharp crested weir is given by the relation

$$Q = \phi H^n$$

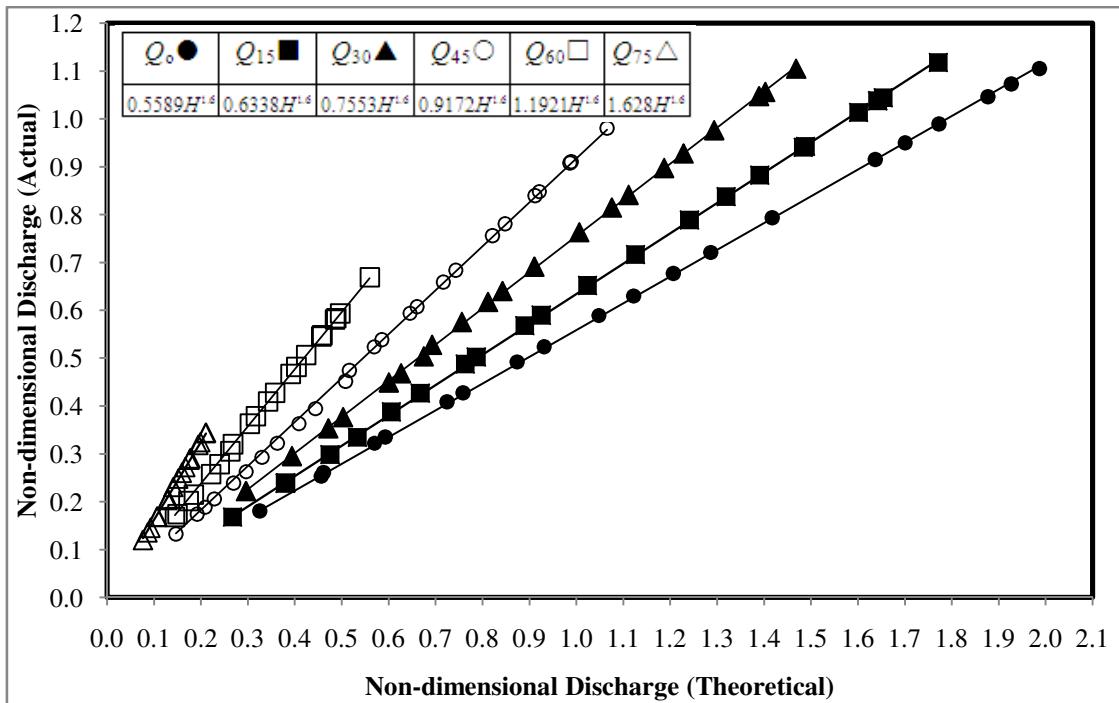


Fig. 4 Variation of Non-dimensional Actual and Theoretical Discharge for various weir inclinations

**Table-2:** Calibrated Head-discharge equations for various angles  $\square$ .

Legend/Method	$Q_0 \bullet$	$Q_{15} \blacksquare$	$Q_{30} \blacktriangle$	$Q_{45} \circ$	$Q_{60} \square$	$Q_{75} \triangle$
SheshaPrakash et.al.	$0.4139H^{1.6}$	$0.4679H^{1.6}$	$0.5558H^{1.6}$	$0.6728H^{1.6}$	$0.8828H^{1.6}$	$1.2383H^{1.6}$
Presented Method	$0.5589H^{1.6}$	$0.6338H^{1.6}$	$0.7553H^{1.6}$	$0.9172H^{1.6}$	$1.1921H^{1.6}$	$1.628H^{1.6}$

In the second phase the obtained 6 weir-discharging coefficients are listed against weir inclinations as radians relatively as  $y_i$  and  $x_i$  and arranged as matrix to develop the model for the present problem, 2<sup>nd</sup>order polynomial curve can be fit to the data and simplifying the equations, we get the final general head-discharge-angle expression for any given trapezoidal weir, of and inclination as under:

$$\begin{bmatrix} 6 & \Sigma x_i & \Sigma x_i^2 \\ \Sigma x_i & \Sigma x_i^2 & \Sigma x_i^3 \\ \Sigma x_i^2 & \Sigma x_i^3 & \Sigma x_i^4 \end{bmatrix} = \begin{bmatrix} \Sigma y_i \\ \Sigma x_i y_i \\ \Sigma x_i^2 y_i \end{bmatrix}$$

Using the data obtained,

$\Sigma x_i = 3.927$	$\Sigma y_i = 5.685$	$\Sigma x_i^3 = 4.037$	$\Sigma x_i^2 y_i = 4.913$
$\Sigma x_i^2 = 3.770$	$\Sigma x_i y_i = 4.661$	$\Sigma x_i^4 = 4.599$	$n = 6$

Substituting the values, we get

$$\begin{bmatrix} 6 & 3.927 & 3.770 \\ 3.927 & 3.770 & 4.037 \\ 3.770 & 4.037 & 4.599 \end{bmatrix} \begin{bmatrix} \alpha^2 \\ \alpha \\ C \end{bmatrix} = \begin{bmatrix} 5.685 \\ 4.661 \\ 4.913 \end{bmatrix}$$

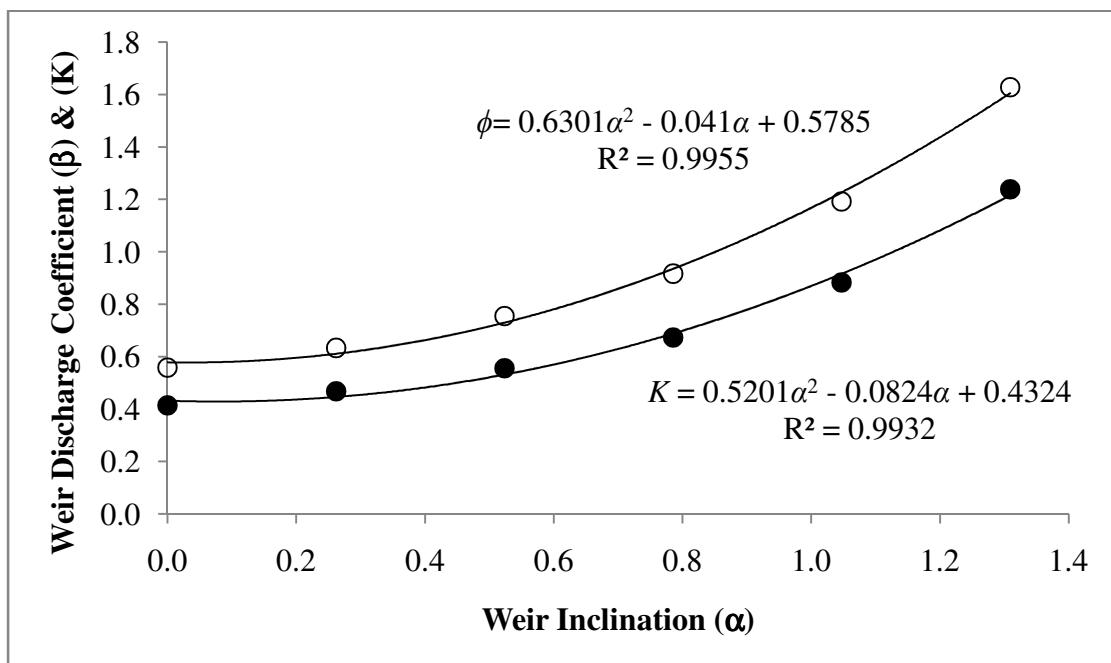
Solving by Gaussian elimination method, we get the quadratic equation as

$$\phi = 0.6301\alpha^2 - 0.041\alpha + 0.5785$$

$$Q = (0.6301\alpha^2 - 0.041\alpha + 0.5785)H^{(1.6)}$$

Where  $Q = \frac{q}{2\sqrt{2g}w^{\frac{5}{2}}}$ ;  $H = \frac{h}{w}$  and  $\alpha$  is in radians

The same is obtained as an Excel curve fit as shown in Fig. 5.



**Fig. 5:** Variation of weir discharge coefficient ( $\phi$ ) and  $K$  with weir inclination ( $\alpha$ )

It can be seen that the equation developed by the model agrees with the one obtained by Excel and further, the regression coefficient in both the cases is exactly unity. The similarity

in the parabolic curve fit between the two methods also corroborates the present method. This improves the credibility of the analysis and practical usage of the notch. Even though obtained discharge-head-inclination equation is complicated, it reduces to simple equation once the  $\alpha$  values are substituted and simplified.

#### Error analysis:

Error analysis is carried out by computing the percentage deviation of the Computed discharge from the actual discharge for various inclinations for method developed by Shesha Prakash et. al and the presented method as shown in Table-3.

**Table-3:** Maximum absolute percentage deviation of Computed to Actual discharge.

<b>Legend/Method</b>	<b>Normal</b>	<b>Angle of inclination with vertical plane in degrees</b>					
		0	15	30	45	60	75
Shesha Prakash et.al.	1.02	1.15	1.24	0.98	1.45	1.40	
Present Research	1.29	1.06	1.10	0.84	2.60	2.74	

#### Discharging capacity:

The discharging capacity of the inclined trapezoidal weir relative to normal position is shown in Fig. 6. **Inclinedweir discharging index** is the ratio of the discharging capacity of an inclined weir to that of normal weir. **Inclinedweir discharging index** for inclined trapezoidal weir for various inclinations with respect to normal position of trapezoidal weir are as plotted in Fig. 5.

$$C_{di} = 1.1273\alpha^2 - 0.0734\alpha + 0.035$$

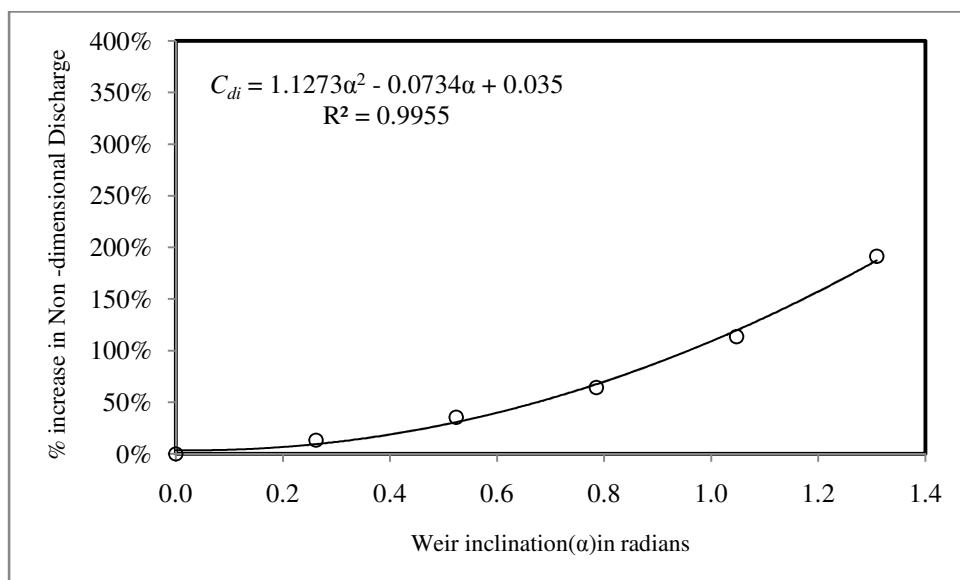


Fig. 6 Variation of Inclined weir discharging index with weir inclination

#### Analysis of afflux:

The Non-Dimensional Head is computed for the Maximum Non-dimensional Discharge for various inclinations of the trapezoidal notch as shown in Table4. It can be seen from Table 4 that the maximum reduction is with  $75^\circ$  inclination.

**Table 4:** Reduction in Afflux and increase in discharging capacity for various inclinations of trapezoidal notch relative to its normal position.

$\alpha$		$Q_{\max} = 2.27 \text{ E-03 LPS}$		$H = \left[ \frac{Q}{\phi} \right]^{\frac{1}{n}}$	%ge Reduction in Afflux	Discharge per unit head	%ge increase in $Q$
Deg	Radians	$\phi$	$n$				
0	0.00	0.559	1.6	0.032	0.0%	0.559	0.0%
15	0.26	0.634	1.6	0.03	6.3%	0.634	13.4%
30	0.52	0.755	1.6	0.027	15.6%	0.755	35.1%
45	0.79	0.917	1.6	0.023	28.1%	0.917	64.1%
60	1.05	1.192	1.6	0.02	37.5%	1.192	113.3%
75	1.31	1.628	1.6	0.016	50.0%	1.628	191.3%

### Conclusions:

Following conclusions were drawn based on the experimental investigation and the subsequent analysis by the authors.

- The advantage of trapezoidal weir is that it can be fixed both to the plumb and also to level as it has crest similar to rectangular weir. Further, the discharging capacity will increase with increase in head similar to V-notch. These two features are important as the weir is to be fixed symmetrical to axis else the computed values will be erroneous.
- The discharging capacity of the weir increases with the increase in inclination of the plane of weir. In particular it is found to increase exponentially from  $30^\circ$  to  $75^\circ$ .
- It is observed from Table3 that the percentage deviation of error in computation of discharge is well within 2% for all angles of inclination of the weir. (except very few data points nearer to the crest and  $60^\circ$  and  $75^\circ$  weir inclination).
- Larger flow area is possible in the inclined weirs relative to the conventional normally positioned weirs. From Table4, it is seen that afflux is found to be about 50.0% with  $75^\circ$  inclination of trapezoidal weir relative to its normal position. The property of increase in **Inclined weir discharging index** with increase in inclination of weir plane can be explored to discharge more water quickly without increasing afflux on upstream side in predesigned canal structure during flood season, without changing the position of the preinstalled weir

(which is practically very difficult). It is found to be 191.3% increase with 75° weir inclination relative to its normal position.

- Due to the simple geometry and ease of construction, inclined trapezoidal weirs find its applications as a simple measuring devices in irrigation, chemical and sanitary engineering for flow measurement and flow control.
- The Mathematical modeling results in a single head discharge inclination equation which can be used for any trapezoidal weir of any desired inclination.

#### **Limitation:**

The experiment can be done with larger discharge in larger channels and the Head Discharge Inclination equation can be improved by using the model.

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