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ρ- SEPARATION AXIOMS WHERE ρ∈{p, q, Q}

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Abstract: Separation axioms play a dominant role in classification of topologies. In this paper, the concepts of p-T_i, q-T_i , and Q-T_i axioms for i = 0, 1, 2 are introduced and their basic properties are studied. Further the concepts of p- regular, p-normal spaces; q- regular, q-normal spaces and Q- regular, Q-normal spaces are also introduced and studied.
Keywords: p-set, q-set, Q-set, ρ- T_0 spaces, ρ- R_1 spaces, ρ- regular and ρ- normal spaces.

1. Introduction

The interior and closure operators in topological spaces play a dominant role in the generalization of closed sets and open sets in topological spaces. These operators have application to Rough set theory, Data mining and Digital image processing. Levine studied the commutativity property of these operators. Mathematicians studied p-sets [5], q-sets [6] and Q-sets [2] that are defined using the interior and closure operators. The purpose of this paper is to study some weaker form of separation axioms using p-sets, q-sets and Q-sets. The basic concepts are given in section 2 and the section 3 is dealt with ρ- T_0 spaces. In section 4, ρ- R_1 spaces are introduced and investigated. ρ- regular and ρ- normal spaces are discussed in section 5.

2. Preliminaries

Throughout this section X is a topological space and A, B are the subsets of X. The notations *cl*A and *int*A denote the closure of A and interior of A in X respectively. Some basic definitions and results are given below.

Definition 2.1

The subset of a topological space A is said to be

(i) A p- set[5] if *cl int* A ⊆ *int c*l A

(ii) A q- set[6] if *cl int* A ⊇ *int c*l A

(iii) A Q-set [2] if *cl int* A= *int c*l A

The properties of p- sets, q- sets, Q-sets are further investigated in [3], [4], [1] respectively.

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3. ρ- T_0 spaces

Definition 3.1

A space X is said to be p-T_0 if for each pair of distinct points x, y of X, there exists a p-set containing one of the points but not the other.

Definition 3.2

A space X is said to be q-T_0 if for each pair of distinct points x, y of X, there exists a q-set containing one of the points but not the other.

Definition 3.3

A space X is said to be Q-T_0 if for each pair of distinct points x, y of X, there exists a Q-set containing one of the points but not the other.

Proposition 3.4

Let X be topological space and U be a subset of X.

(i) If $x \in X$, U is a p-set with $x \in U$ then p-$cl(x) \subseteq U$.

(ii) If $x \in X$, U is a q-set with $x \in U$ then q-$cl(x) \subseteq U$.

(iii) If $x \in X$, U is a Q-set with $x \in U$ then Q-$cl(x) \subseteq U$.

Proposition 3.5

(i) $x \in$ p-$cl(A) \Leftrightarrow U \cap A \neq \varnothing$ for every p-set U containing x.

(ii) $x \in$ q-$cl(A) \Leftrightarrow U \cap A \neq \varnothing$ for every q-set U containing x.

(iii) $x \in$ Q-$cl(A) \Leftrightarrow U \cap A \neq \varnothing$ for every Q-set U containing x.

Proposition 3.6

A space X is p-T_0 if and only if for each pair of distinct points x,y of X, $y \notin$ p-$cl(\{x\})$

Proof

Suppose X is a p-T_0 space. Let $x, y \in X$ such that $x \neq y$. By Definition 3.1, there exists a p-set U containing x but not y that is say $x \in U$ and $y \notin U$. This shows that $y \notin$ p-$cl(\{x\})$. Conversely, let $x, y \in X$, $x \neq y$ such that $y \notin$ p-$cl(\{x\})$. Then using Proposition 3.5, there is a p-set U such that $x \in U$ and $y \notin U$. This proves that X is p-T_0.

Theorem 3.7

Let X be topological space. Then the following are equivalent.

(i) X is p-T_0.

(ii) For each pair of distinct points x, y of X, p-$cl(x) \neq$ p-$cl(y)$.

(iii) For each pair of distinct points x,y of X, there are p-sets U and V in X such that $x \in U$, $y \notin U$ and $y \in V$ and $x \notin V$.

(iv) For each pair of distinct points x, y of X, there are p-sets U and V in X such that x∈ U, y ∉ U and y∈V and x ∉ V.

Proof

Suppose X is p-T_0. Let x, y ∈ X with x ≠ y. Then using proposition 3.6, y ∉p-c*l*(x) and x ∉ p-c*l*(y). This proves that p-c*l*(x) ≠ p-c*l*(y). This proves (i) ⇒ (ii). Now let p-c*l*(x) ≠ p-c*l*(y) for each pair x,y of distinct points of X. Suppose y ∈ p-c*l*(x). Then by using Proposition 3.4,

p-c*l*(y) ⊆ p-c*l*(x). This shows that y ∉ p-c*l*(x). Therefore by using proposition 3.6, X is p-T_0. This proves (ii) ⇒ (i).

Suppose X is p-T_0. Let x, y∈X with x≠ y. Then using Definition 3.1, there is a p-set U with x∈U and y∉U. Since V =X− U is a p-set, y∈V and x ∉ V. This proves (i) ⇔ (iii)⇔ (iv).

Proposition 3.8

A space X is q-T_0 if and only if for each pair of distinct points x,y of X, y∉ q-c*l*({x})

Proof

Suppose X is a q-T_0 space. Let x, y ∈ X such that x ≠ y. By Definition 3.2, there exists a q-set U containing x but not y that is say x∈ U and y∉ U. This shows that y∉ q-c*l*({x}). Conversely let x, y ∈ X, x ≠ y such that y∉ q-c*l*({x}). Then there is a q-set U such that x∈ U and y∉ U. This proves that X is q-T_0.

Theorem 3.9

Let X be topological space. Then the following are equivalent.

(i) X is q-T_0

(ii) For each pair of distinct points x, y of X , q-c*l*(x) ≠ q-c*l*(y).

(iii) For each pair of distinct points x,y of X, there are q-sets U and V in X such that x∈ U, y ∉ U and y∈V and x ∉ V.

(iv) For each pair of distinct points x,y of X, there are disjoint q-sets U and V in X such that x∈ U and y∈V.

Proof

Suppose X is q-T_0. Let x, y ∈ X with x≠ y. Then using Proposition 3.8, y ∉q-c*l*(x) and x ∉ q-c*l*(y). This proves that q-c*l*(x) ≠ q-c*l*(y). This proves (i)⇒ (ii). Now let q-c*l*(x) ≠ q-c*l*(y) for each pair x,y of distinct points of X. Suppose y ∈q-c*l*(x). Then by using Proposition 3.4, q-cl(y) $\subseteq$ q-cl(x). This shows that y $\notin$ q-cl(x). Therefore by using Proposition 3.8, X is q-T_0. This proves (ii)$\Rightarrow$ (i).

Suppose X is q-T_0. Let x, y$\in$X with x$\neq$ y. Then using Definition 3.2, there is a q-set U with x$\in$U and y$\notin$U. Since V =X− U is a q-set, y$\in$V and x $\notin$ V. This proves (i) $\Leftrightarrow$ (iii)$\Leftrightarrow$ (iv).

Proposition 3.10

A space X is Q-T_0 if and only if for each pair of distinct points x,y of X, y$\notin$ Q-cl({x})

Proof

Suppose X is a Q-T_0 space. Let x, y $\in$ X such that x $\neq$ y. By Definition 3.3, there exists a Q-set U containing x but not y that is say x$\in$ U and y$\notin$ U. This shows that y$\notin$ Q-cl({x}). Conversely, let x, y $\in$ X, x $\neq$ y such that y$\notin$ Q-cl({x}). Then there is a Q-set U such that x$\in$ U and y$\notin$ U. This proves that X is Q-T_0.

Theorem 3.11

Let X be topological space. Then the following are equivalent.

(i) X is Q-T_0

(ii) For each pair of distinct points x, y of X , Q-cl(x) $\neq$ Q-cl(y).

(iii) For each pair of distinct points x,y of X, there are Q-sets U and V in X such that x$\in$ U, y $\notin$ U and y$\in$V and x $\notin$ V.

(iv) For each pair of distinct points x,y of X, there are disjoint Q-sets U and V in X such that x$\in$ U and y$\in$V.

Proof

Suppose X is Q-T_0. Let x, y $\in$ X with x$\neq$ y. Then using Proposition 3.10, y $\notin$ Q-cl(x) and x $\notin$ Q-cl(y). This proves that Q-cl(x) $\neq$ Q-cl(y). This proves (i) $\Rightarrow$ (ii). Now let Q-cl(x) $\neq$ Q-cl(y) for each pair x,y of distinct points of X. Suppose y $\in$Q-cl(x). Then by using Proposition 3.4, Q-cl(y) $\subseteq$ Q-cl(x). This shows that y $\notin$ Q-cl(x). Therefore by using Proposition 3.10, X is Q-T_0. This proves (ii)$\Rightarrow$ (i).

Suppose X is Q-T_0. Let x, y$\in$X with x$\neq$ y. Then using Definition 3.3, there is a Q-set U with x$\in$U and y$\notin$U. Since V =X− U is a Q-set, y$\in$V and x $\notin$ V. This proves (i) $\Leftrightarrow$ (iii)$\Leftrightarrow$ (iv).

4. ρ- R_1 spaces

Definition 4.1

A space X is said to be p- R_1 if for each pair of distinct points x,y of X with p-c*l*(x) ∩ p-c*l*(y) = ∅ there are disjoint p-sets U and V in X such that p-c*l*(x) ⊆ U and p-c*l*(y)⊆ V.

Definition 4.2

A space X is said to be q- R_1 if for each pair of distinct points x,y of X with q-c*l*(x) ∩ q-c*l*(y) = ∅ there are disjoint q-sets U and V in X such that q-c*l*(x) ⊆ U and q-c*l*(y) ⊆ V.

Definition 4.3

A space X is said to be Q- R_1 if for each pair of distinct points x,y of X with Q-c*l*(x) ∩ Q-c*l*(y) = ∅ there are disjoint Q-sets U and V in X such that Q-c*l*(x) ⊆ U and Q-c*l*(y)⊆ V.

Theorem 4.1

(i) If X is p-T_0 then it is p-R_1. The converse is not true.

(ii) If X is q-T_0 then it is q-R_1. The converse is not true.

(iii) If X is Q-T_0 then it is Q-R_1. The converse is not true.

5 . ρ- regular and ρ- normal spaces

Definition 5.1

X is p-regular if for x ∈ X and a closed set F ⊂ X with x∉ F, their exist p-sets U and V such that x∈ U, F⊂ V and U ∩ V = ∅.

Definition 5.2

X is q-regular if for every x ∈ X and a closed set F ⊂ X with x∉ F, their exist q-sets U and V such that x∈U, F⊂ V and U ∩ V = ∅.

Definition 5.3

X is Q-regular if for every x ∈ X and a closed set F ⊂ X with x∉ F, their exist Q-sets U and V such that x∈U, F⊂ V and U ∩ V = ∅.

The proof for the next three proposition is routine work.

Proposition 5.4

Let X be a topological space. Then the following are equivalent.

(i) X is p-regular.

(ii) for every x ∈ X and for every closed set F with x∉ F, there exists a p-set V such that x∉V, F⊂ V.

(iii) for every $x \in X$ and for every open set G with $x \in G$ there exists a p-set V such that $x \in V \subset G$.

Proposition 5.5

Let X be a topological space. Then the following are equivalent.

(i) X is q-regular.

(ii) for every $x \in X$ and for every closed set F with $x \notin F$, there exists a q-set V such that $x \notin V$, $F \subset V$.

(iii) for every $x \in X$ and for every open set G with $x \in G$ there exists a q-set V such that $x \in V \subset G$.

Proposition 5.6

Let X be a topological space. Then the following are equivalent.

(i) X is Q-regular.

(ii) for every $x \in X$ and for every closed set F with $x \notin F$, there exists a Q-set V such that $x \notin V$, $F \subset V$.

(iii) for every $x \in X$ and for every open set G with $x \in G$ there exists a Q-set V such that $x \in V \subset G$.

Definition 5.7

X is p-normal if for any two disjoint closed sets A and B in X , there exist disjoint p-sets U and V such that $A \subseteq U$, $B \subseteq V$.

Definition 5.8

X is q-normal if for any two disjoint closed sets A and B in X , there exist disjoint q-sets U and V such that $A \subseteq U$, $B \subseteq V$.

Definition 5.9

X is Q-normal if for any two disjoint closed sets A and B in X , there exist disjoint Q-sets U and V such that $A \subseteq U$, $B \subseteq V$.

The next three proposition can be easily established.

Proposition 5.10

Let X be a topological space. Then the following are equivalent.

(iv) X is p-normal.

(v) for any two disjoint closed sets A and B in X there exists a p-set V such that $A \subseteq V$ and $B \subset X - V$.

(vi) for any two disjoint open sets A and B in X there exists a p-set V such that $A \subseteq V$ and $B \subset X-V$.

Proposition 5.11

Let X be a topological space. Then the following are equivalent.

(i) X is q-normal.

(ii) for any two disjoint closed sets A and B in X there exists a q-set V such that $A \subseteq V$ and $B \subset X-V$.

(iii) for any two disjoint open sets A and B in X there exists a q-set V such that $A \subseteq V$ and $B \subset X-V$.

Proposition 5.12

Let X be a topological space. Then the following are equivalent.

(i) X is Q-normal.

(ii) for any two disjoint closed sets A and B in X there exists a Q-set V such that $A \subseteq V$ and $B \subset X-V$.

(iii) for any two disjoint open sets A and B in X there exists a Q-set V such that $A \subseteq V$ and $B \subset X-V$.

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